<u>Problem Set 1</u>

CS 518: Quantum Computing

Out: Week of 09/09/2015
Due: The beginning of the class in the week of 09/21/2015
Total Points: 80

Solutions should be in a ***typed-up*** *format by using LaTeX or microsoft word. However, if you choose to write up by hand, please have the solution neatly written. Remember, your goal is to communicate. Full credit will be given* ***only*** *to the correct solution which is described clearly. Convoluted and obtuse descriptions might receive low marks, even when they are correct. Also, aim for concise solutions, as it will save you time spent on write-ups, and also help you conceptualize the key idea of the problem.*

The kets $|+\rangle$, $|-\rangle$, $|0\rangle$, and $|1\rangle$ are defined as follows:

$$|0\rangle := \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |1\rangle := \begin{pmatrix} 0 \\ 1 \end{pmatrix}, |+\rangle := \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle, |-\rangle := \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Problem 1 (10pts)

Please watch https://www.youtube.com/watch?v=x6eR2vjdddY

Problem 2 (10pts)

Exercise 3.3.1 Consider the 2-qubit state

$$|\psi\rangle = \tfrac{1}{\sqrt{2}}|0\rangle|0\rangle + \tfrac{1}{\sqrt{2}}|1\rangle|1\rangle. \tag{3.3.2}$$

Show that this state is entangled by proving that there are no possible values $\alpha_0, \alpha_1, \beta_0, \beta_1$ such that

$$|\psi\rangle = \big(\alpha_0|0\rangle + \alpha_1|1\rangle\big)\big(\beta_0|0\rangle + \beta_1|1\rangle\big). \tag{3.3.3}$$

Problem 3 (15pts)

Exercise 5.2.1 Prove that

$$|\psi\rangle|\beta_{00}\rangle = \tfrac{1}{2}|\beta_{00}\rangle|\psi\rangle + \tfrac{1}{2}|\beta_{01}\rangle(X|\psi\rangle) + \tfrac{1}{2}|\beta_{10}\rangle(Z|\psi\rangle) + \tfrac{1}{2}|\beta_{11}\rangle(XZ|\psi\rangle). \tag{5.2.8}$$

Problem 4

(12 points) Which of the following sets are orthonormal bases (ONB) of $\mathbb{C}^2$.

(a) (2 points) $\{|+\rangle, |-\rangle\}$

(b) (2 points) $\{|0\rangle, |1\rangle\}$

(c) (2 points) $\{|0\rangle - |1\rangle, |0\rangle + |1\rangle\}$

(d) (2 points) $\{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -i\end{pmatrix}\}$

(e) (2 points) $\{\frac{1}{\sqrt{2}}\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}\sin\theta\\ -\cos\theta\end{pmatrix}\}$

(f) (2 points) $\{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -1\end{pmatrix}\}$

Problem 5

(9 points) Let $|\psi\rangle, |\varphi\rangle$ be an orthonormal basis in the Hilbert space $\mathbb{C}^2$.
Let $A := |\psi\rangle\langle\psi| + |\varphi\rangle\langle\varphi|$.

Find the matrix representation of A with respect to the basis $\{|0\rangle, |1\rangle\}$ where $|\psi\rangle$ and $|\varphi\rangle$ are as follows:

(a) (3 points) $|\psi\rangle := \begin{pmatrix}1\\ 0\end{pmatrix}, |\varphi\rangle := \begin{pmatrix}0\\ 1\end{pmatrix}$

(b) (3 points) $|\psi\rangle := \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix}, |\varphi\rangle := \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -1\end{pmatrix}$

(c) (3 points) $|\psi\rangle := \begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}, |\varphi\rangle := \begin{pmatrix}\sin\theta\\ -\cos\theta\end{pmatrix}$

Problem 6 (12pts)

$$T = |0\rangle\langle 0| \otimes (|+\rangle\langle +| + |-\rangle\langle -|) + |1\rangle\langle 1| \otimes (|+\rangle\langle +| - |-\rangle\langle -|).$$

Show that T and $CNOT$ are equal.

Problem 7

(12 points) Compute:

(a) (4 points) $\langle 10| + -\rangle$ and $\langle 1|+\rangle\langle 0|-\rangle$

(b) (4 points) $\langle 01| - +\rangle$ and $\langle 0|-\rangle\langle 1|+\rangle$

(c) (4 points) Let $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$ and $|\phi\rangle = |\phi_1\rangle \otimes |\phi_2\rangle$, where $|\psi_1\rangle, |\psi_2\rangle, |\phi_1\rangle$, and $|\phi_2\rangle$ are arbitrary vectors in $\mathbb{C}^2$. Show that $\langle\psi|\phi\rangle = \langle\psi_1|\phi_1\rangle\langle\psi_2|\phi_2\rangle$.