MAT 115: Problem Set 2

Section: MW 4-5:50 pm

Due: 10/5/2015

Problem 1 Propositional Logic

(a) Is the statement form $((\sim p \wedge q) \wedge (q \vee r)) \wedge \sim q \wedge r$ a tautology, contradiction or neither?
(b) Is $(p \wedge \sim q) \wedge (\sim p \vee q) \wedge r$ a tautology, contradiction or neither?
(c) Given $(\sim p \vee q) \Rightarrow (r \vee \sim q)$, rewrite it as statement form using only $\sim$ and $\wedge$.
(d) Please use the definition of $p \Rightarrow q$ as shown on page Lo-5 on the lecture note to show $p \Rightarrow q$ is equivalent to $\sim p \vee q$.

Problem 2 Predicate Logic

Which of the following statements are true, which are false? ($\exists!$ means "there exists exactly one) If false, please show counter example.
(a) $\exists! x \in \mathbb{Z} \ni 1/x \in \mathbb{Z}$.
(b) $\forall x \in \mathbb{R}, \exists! y \in \mathbb{R} \ni x + y = 0$
(c) $\forall m \in \mathbb{N}, \exists n \geq m, n$ even, $\exists p, q \in \mathbb{P}, n = p + q$.
(d) $\forall m \in \mathbb{N}, \exists n \geq m, n$ odd, $\exists p, q \in \mathbb{P}, n = p + q$.
(e) $D = \{1, 3, 4, 5, 9, 121, 169, 196, 225\}, S(x) = (\sqrt{x} \in \mathbb{Z} \wedge \sqrt{x} \in \mathbb{P})$. Let $S = \{x \in D | S(x)\}$. Please show the elements inside the set S.

Problem 3 Ordering Sets

Let $A = \{w, x, y, z\}, B = \{1, 2\}, C = \{\alpha, \beta\}$. Please show, by use of lex order, the result from the following product: (a) $A \times B \times C$ (b) $(A \times B) \times C$

Problem 4 Sets Algebraic Rules

Please prove by use of set algebraic rules for the follwoing sub problems:
(a) $(P - Q) \cap (R - Q) = (P \cap R) - Q$.
(b) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

Problem 5 Practice Problems

For practice only. You do not have to turn in the solution.
Unit Lo: 1.10, 1.19, 2.13, 2.19.11, 1.1
Unit SF: 1.1, 1.6, 1.11, 1.14