MAT 115: Problem Set 3

Section: MW 4-5:50 pm

Due: 10/28/2015

Problem 1 Power Set

Let $A = \{1, 3, 5, 6, 7\}$ and suppose B is the power set of A, i.e. $B = \mathcal{P}(A)$.
(a) Please list the elements (subsets of A) in B.
(b) Let $C = \mathcal{P}(B)$. How many elements (subsets) are there in C?

Problem 2 Binomial Recursion

Please show

$$\begin{pmatrix} n \\ k \end{pmatrix} = \begin{pmatrix} n-1 \\ k-1 \end{pmatrix} + \begin{pmatrix} n-1 \\ k \end{pmatrix}$$

Problem 3 Induction Proof

Please prove the following by using induction proof. Make sure you mark the base case, hypothesis and the induction step clearly.
(a) Please show

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

(b) Please show

$$\sum_{i=1}^{n} (2i+1) = n(n+2)$$

Problem 4 Rational Number

Sow that $\sqrt{5} + 3$ is not rational, provided that we know $\sqrt{5}$ is not rational.

Problem 5 Other Proof Techniques: [Counterexample, Contrapositive, Contradiction, Case by case]

Prove the statement if true, otherwise find a counterexample. When you prove, if you see **if and only if** in the statement, you are supposed to prove for both directions.
(a) $\forall m, n \in \mathbb{Z}, m^3 - n^3$ is even if and only if $m - n$ is even.
(b) $\forall n \in \mathbb{Z}, n^2 - n + 2$ is even.
(c) For all distinct positive integers m and n, both m and n are perfect squares if and only if $m + 2m^{1/2}n^{1/2} + n$ is a perfect square.
(d) For all distinct positive integers m and n, both m and n are perfect squares if and only if $m^{1/2}n^{1/2}$ is an integer.

Problem 6 Practice Problems

For practice only. You do not have to turn in the solution.
Unit SF: 1.21
Unit NT: 1.2, 1.3, 1.4(b), 1.13, 1.14, 1.23, 1.27, 1.28