MAT 115: Problem Set 5

Section: MW 4-5:50 pm

Due Date: 12/09/2015

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Group ID:

Score: /

Problem 1 Equivalent Classes and Partition

Let $S = \{a, b, c, d\}$. What are the possible equivalence relations on S?

Problem 2 Proof: Equvalence relation

Let $\mathbb{R}$ be the real numbers. Define $x \equiv y$ to be related if $x - y \in \mathbb{Z}$. Show that $\equiv$ is an equivalence relation on $\mathbb{R}$. You need to show this relation has the following properties:
(a) Reflexivity:

(b) Symmetry:

(c) Transitivity:

Problem 3 Pigeonhole Concept

Let $t_1, t_2, \cdots t_n$ be n **distinct** integers. Show that either $n|t_k$ for some k or $n|(t_i - t_j)$ for some $i \neq j$ (Hint: classsify those $t_1, \cdots t_n$ numbers by a modulo n function).

Problem 4 Relation to Partition $\Longleftrightarrow$ Partition to Relation

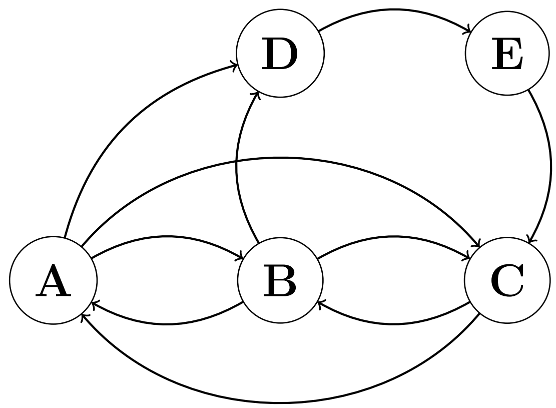
Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 4), (4, 1), (2, 3), (3, 2)\}$.
(a) Is R an equivalence relation? (need to verify those three properties)

(b) What are the equivalent classes (partitions) of A

(c) $A = \{1, 2, 3, 4, 5, 6\}$ and it can be partitioned by R_2 such that the partitions are $P_1 = \{1, 4\}, P_2 = \{2\}$ and $P_3 = \{3, 5, 6\}$. Please write out the relation R_2.

Problem 5 Graph: Definition and Cycles

Suppose you are given the following **directed** graph $G = (V, E)$.



(a) Please write out G based on the graph above.

(b) Find 3 cycles within the graph. Each cycle contains at least 4 distinct vertices.

Problem 6 Graph: Edges

For a graph $G = (V, E)$, let $d(v)$ be the degree of the vertic $v \in V$. Prove that
(a) $\sum_{v \in V} d(v) = 2|E|$, an even number.

(b) Conclude that number of vertices v for which $d(v)$ is odd is even.

Problem 7 Additional Material

Please read the following examples (sovled) in the pdf uploaded on blackboard to help you better understand some definitions and solving techniques.

(a) Example 1 on page EO-1 [notice, they show the partitions (equivalence classes). If you are asked to show the equivalence relation, make sure the 3 properties (symmetric, reflexive, transitive) hold.

(b) Example 6 on page EO-4.

(c) Example 9 on page EO-6

(d) If you are still a bit rusty regarding cycle notations for functions, please read from page Fn-1 to page Fn-10. That would help.