MAT 115: Finite Math for Computer Science
Problem Set 2

Due: 10/03/2016

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Score: /100 +   /15 (bonus)

Problem 1   Circuit

Please explain why the full adder (on BF-19) has OR at the end, instead of another half adder. Also, is it OK to replace that OR gate with XOR gate? Why?

Problem 2   Propositional Logic

Is the statement form $((\sim p \wedge q) \wedge (q \vee r)) \wedge \sim q \wedge r$ a tautology, contradiction or neither? Please use algebraic rules.

Problem 3 Propositional Logic

Is $(p\wedge \sim q)\wedge (\sim p\vee q)\wedge r$ a tautology, contradiction or neither? Please use algebraic rules.

Problem 4   Predicate Logic

Start with the statement, "$\forall n \in \mathbb{N}$, if n^2 is even, then n is even". Form the contrapositive, converse, and inverse of the statement. Which statements are true? [Hint: Let p be defined as $\forall n \in \mathbb{N}$, if n^2 is even. Let q be defined as n is even. The statement is $p \to q$.]

Problem 5   Predicate Logic

$D = \{1, 3, 4, 5, 9, 121, 169, 196, 225, 289\}, S(x) = (\sqrt{x} \in \mathbb{Z} \wedge \sqrt{x} + 2 \in \mathbb{P})$ where $\mathbb{P}$ is the set of prime numbers. Let truth set $T = \{x \in D | S(x)\}$. Please show the elements inside the set T.

Problem 6   Proof

Please show that for any integer m and n, $m^3 - n^3$ is even **if only if** $m - n$ is even.
(a) Given any integer m and n, if $m^3 - n^3$ is even, then $m - n$ is even

(b) Given any integer m and n, if $m - n$ is even, then $m^3 - n^3$ is even

Problem 7   Proof

Prove or disprove the following:
(a) The product of two integers is even **if and only if** at least one of them is even

(b) The product of two integers is odd **if and only if** at least oe of them is odd

Problem 8　　Floor Ceiling Functions

Please compute the following:

(a) $\lceil (\lfloor 3.85 \rfloor * \lceil -6.3 \rceil + 2.5) \rceil * 3$

(b) $(\lfloor -3.4 \rfloor * \lceil 5.3 \rceil) + \lceil (2.5 * 3.3) \rceil$

Problem 9   Modulo Functions

Prove that for all integers $n > 1$, $n^2 - 3 \neq 0 \pmod 4$. [Hint: Do by cases]

Problem 10   Simple Induction

Please use induction proof method to show that $\sum_{i=1}^{n} i^2 + i$ has a closed form as $\frac{n(n+1)(n+2)}{3}$

Base case:

Hypothesis:

Induction:

Problem 11   Bonus: Mersenne Number (15pts)

We talked about Mersenne Number and Mersenne Prime Number in class. Please reserach on the internet and answer the following:

(a) List 2 applications of prime numbers in computer science

(b) What is the current biggest prime number? Is it a Mersenne Prime?

(c) Based the exercise 2.21 on Lo-22 and the answer you have in (b), what is the current largest perfect number

Problem 12   Practice Problems

For practice only. You do not have to turn in the solution.
Unit Lo: 1.10, 1.19, 2.13, 2.19.11, 1.1
Unit NT: 1.6(a)(b), 1.14, 1.19, 1.25(a), 1.28