MAT 115: Finite Math for Computer Science
Problem Set 6

Out: 11/16/16 Due: 11/28/2016

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Score: **/75 + 10**

Problem 1 Pigeonhole Concept (10pts)

Let $t_1, t_2, \cdots t_n$ be n **distinct** integers. Show that either $n|t_k$ for some k or $n|(t_i - t_j)$ for some $i \neq j$ (Hint: classsify those $t_1, \cdots t_n$ numbers by a modulo n function).

Problem 2 Combinatorics (4 + 4 pts)

You are controlling a robot. The task is given that your robot is set at coordinate $(1, 2)$ and your robot has to move to coordinate (9,12). Let say each time your robot can only move up by 1 in Y axis or right by one in the X axis. Let us assume each move is of the same cost, therefore, all possible paths are of the same total cost. Please show how many paths there are for your robot to move from

(a) $(1, 2)$ to $(9, 12)$

(b) $(1, 2)$ to $(9, 12)$ but must pass $(5, 7)$ in the path.

Problem 3 Permutation: With Repetition ($3 \times 7 = 21$ pts)

We work with the ordinary alphabet of 26-letters. Please solve the following:
(a) Define a 5-letter word to be any list of 5 letters that contains *at least* one of the vowels A, E, I, O and U. How many 5-letter words are there?

(b) We can solve (a) in one single step but we can do it in 6 steps. Here are the steps:
(b-1) How many 5-letter words with exactly 1 vowel

(b-2) How many 5-letter words with exactly 2 vowels

(b-3) How many 5-letter words with exactly 3 vowels

(b-4) How many 5-letter words with exactly 4 vowels

(b-5) How many 5-letter words with exactly 5 vowels

(b-6) Your sum from b-1 till b-5 is? The result should be equivalent to (a)

Problem 4 Permutation: With NO Repetition ($3 \times 7 = 21$ pts)

We work with the ordinary alphabet of 26-letters. Please solve the following:
(a) Define a 5-letter word to be any list of 5 letters that contains *at least* one of the vowels A, E, I, O and U. How many 5-letter words are there?

(b) We can solve (a) in one single step but we can do it in 6 steps. Here are the steps:
(b-1) How many 5-letter words with exactly 1 vowel

(b-2) How many 5-letter words with exactly 2 vowels

(b-3) How many 5-letter words with exactly 3 vowels

(b-4) How many 5-letter words with exactly 4 vowels

(b-5) How many 5-letter words with exactly 5 vowels

(b-6) Your sum from b-1 till b-5 is? The result should be equivalent to (a)

Problem 5 Permutation: With and Without Repetition ($5 \times 3 = 15$ pts)

We are interested in forming 3 letter words using the letters in LITTLEST. For the prpose of the problem, a word is any list of letters. Please answer the following:
(a) How many words can be made with no repeated letters?

(b) How many words can be made with unlimited repetition allowed?

(c) How many words can be made if repeats are allowed but no letter can be used more than it appears in LITTLEST?

Problem 6 Bonus: 4 + 6 pts

In the two-sum property problem, we are given $S = \{1, 2, \cdots, 16\}$ and we are asked to find the the smallest number k such that a subset, T, of S is of k distinct elements, of S and this subset T must have the two-sum property. In the computation, we know we are computing the number of inputs (possible subsets of T) and the number of outputs (possible sums for those subsets). We want to establish the fact that the cardinality of input is greater than the cardinality of output will make the two-sum property possible for T. Please explain why

(a) The number of possible subsets as input is $2^k - 2$

(b) Please show the steps that the number of possible sums is $16(k-1) - \frac{(k-1)(k-2)}{2}$