MAT 115: Finite Math for Computer Science
Problem Set 7

Out: 11/28/16 Due: 12/07/2016

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Score: **/75**

Problem 1 Probability (3× 3 × 4 = 36 pts)

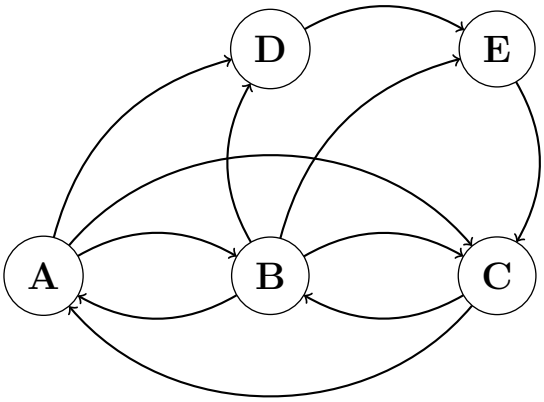
An urn contains ten labeled balls, labels 1,2, $\cdots$, 10.
(a) Two balls are drawn together. What is the sample space? What is the probability that the sum of the labels on the balls is odd? What is the probability that the sum of the labels on the balls is 7?

(b)Two balls are drawn one after the other without replacement. What is the sample space? What is the probability that the sum of the labels on the balls is odd? What is the probability that the sum of the labels on the balls is 9?

(c) Two balls are drawn one after the other with replacement. What is the sample space? What is the probability that the sum of the labels on the balls is odd? What is the probability that the sum of the labels on the balls is 9?

Problem 2 Graph: Definition and Cycles: 5 + 2 × 3 pts

Suppose you are given the following **directed** graph $G = (V, E)$.



(a) Please write out G based on the graph above.

(b) Find 3 cycles within the graph. Each cycle contains at least 4 distinct vertices.

Problem 3 Striling Number: 5 + 5 + 3 + 3 pts

For $n > 0$, the Stirling number of the 2nd kind is $S(n, k)$
(a) Prove that $S(n, n-1) = C(n, 2)$

(b) $S(n, 2) = 2^{n-1} - 1$

(c) What is Sitrling number of the first kind? What are the applications for Stirling number of the 2nd kind?

Problem 4 Graph: Edges: 6 + 6 pts

For a graph $G = (V, E)$, let $d(v)$ be the degree of the vertices $v \in V$. Prove that
(a) $\sum_{v \in V} d(v) = 2|E|$, an even number.

(b) Conclude that number of vertices v for which $d(v)$ is odd is even.

Problem 5 Additional Material (no need to turn in)

Please read the following examples (solved) in the pdf uploaded on blackboard to help you better understand some definitions and solving techniques.

Problem Example 1, 3, 4, 7, 10, 14, 17, 19, on page CL-41, CL42 and CL43