CS 528: Quantum Computing

Take Home Final Exam

Due Date: 12/14/2017

Instructions:
You can discuss within your group regarding how to solve the problems but you must write your own solution **individually**. Plagiarism immediately results in a zero score in this take home final exam. I leave plenty of space on each page for your computation. One extra blank sheet is also attached at the end of this exam if you need more computation space. If your answer is incorrect but you show the computation process, then partial credits will be given.

First Name:

Last Name:

Group ID:

Score: **/140**

Problem 1 General Pauli and QFT: 5 + 10 + 5=20 pts

For $n > 0$, let X_n and Z_n be n-qubit unitary operators such that for all $x \in \mathbb{Z}_{2^n}$, we have

$$X_n|x\rangle = |(x+1)\%2^n\rangle, \qquad Z_n|x\rangle = e_n(x)|x\rangle$$

where % is the modulo function and $e_n(x) = e^{2\pi i x/2^n}$

(a) What are $X_n^* Z_n X_n$ and $Z_n^* X_n Z_n$? (Show how they act on x)((See Problem IV in Final on SF's note)

(b) Draw an n-qubit quantum circuit that implements Z_n using only single-qubit conditional phase shift gates $P(\theta)$ for varous θ.

(c) What are the eigevalues and eigenvectors of X_n?

Problem 2 EPR Pair + Teleportation : 5 +5 + 5 pts

(a) Given an EPR pair, draw the circuit to reverse the pair back to a pair of $|0\rangle$

(b) Please show that an EPR pair cannot be written as a tensor product of two 1-qubit states. An EPR pair is defined as follows: $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

(c) Simply describe the teleportation protocol and repeat the computation

Problem 3 Shor's Algorithm: 5 + 5 + 5 pts

In Shor's algorithm, we have two registers, let say Reg1 and Reg2, and we want to factorize the number N. Reg1 consists of l qubits and that of Reg2 is $n = \lceil \log N \rceil$. Suppose we want to use Shor's algorithm to find the period of the function $f(x) = 5^x$ mod 14 by using a Fourier transform over $q = 256$ (in another word, Reg1 has 8 qubits and it is obvious $N^2 = 14^2 < q = 2^8 = 256 \leq 2N^2$).
(a) What is the order ?

(b) Write down the state (superposition) after we perform the first measurement on the 2nd register (and suppose we get $|9\rangle$ in the 2nd register).

(c) Write down the state (superposition) right before we perform the 2nd measurement on the 1st register.

Problem 4 Grover's Algorithm: 5 + 5 pts

In $\{|G\rangle, |B\rangle\}$ basis, we may write the Grover iteration as

$$G = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

(b-1) Given sample space Ω where $|\Omega| = 2^8$ and let $Sx = \{x|f(x) = 1 \wedge x \in \Omega\}$. Let say $|S_x| = 6$ and you run Grover in order to find the possible solutions. Please write out the Grover operator in real numbers. [Hint: Be careful as each Grover roation rotates by 2*the original degree]

(b-2) What is the number of required invocations of Grover operator? (Be precise to the **2nd digit** after the decimal point).

Problem 5 Deutsch Algorithm: 15pts

In the Deutsch algorithm, suppose we did not prepare $\frac{|0\rangle - |1\rangle}{\sqrt{2}}$ but $|\mathbf{0}\rangle$ instead in the target qubit and we just run the same circuit on that configuration. Please compute and explain what happens at the end of measurement. Furthermore, please conclude the probability that we get the **wrong** answer.

Problem 6 Simon's Algorithm + Concept: 10 + 5 pts

In Simon's algorithm ($f : \{0,1\}^n \to \{0,1\}^n$, $f(x) = f(y)$ iff $x = y$ or $x = s \oplus y$), one of the key ideas is to find the bit strings z that are perpendicular to the hidden string s. That is to say we are seeking z in the set $s^\perp$ where $s^\perp = \{z \in \{0,1\}^n | s \cdot z = 0 \pmod 2\}$. Let $x, y, \in \{0,1\}^n$ and let $s = x \oplus y$. Please show the following:

(a) $H^{\otimes n}[\frac{1}{\sqrt{2}}(|x\rangle + |y\rangle)] = \frac{1}{\sqrt{2^{n-1}}} \sum_{z \in s^\perp} (-1)^{x \cdot z} |z\rangle$

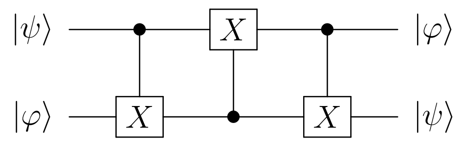
(b) There is some classical post-processing in the Simon's algorithms because we need to process the samples collected at the end of each run of this algorithm (i.e. the measurement on the first register). Please explain why the classical process is necessary and what techniques are used in order to find s.

Problem 7 SWAP: 4*5=20 pts

(a-1) Show that if V is any unitary operator, then there exists a (not necessary unique) unitary U such that $U^2 = V$. [See Problem III in the Final of SF's note].

(a-2) Why did we say not necessary unique in (a-1)?

(b) Via matrix multiplication, show that the following circuit implements **Swap** operator. X is the NOT gate.



(c) Find a two-qubit U such that $U^2 = SWAP$

Problem 8 Topological QW on IBM-Q: (2+8+10) pts

Please find this paper **Physical realization of topological quantum walks on IBM-Q and beyond** on arXiv https://arxiv.org/pdf/1710.03615.pdf.

(a) Is there any problem in equation (4) in terms of dimensionality?

(b) Please verify equation (9) $L^{\pm}|\bar{k}\rangle_\omega = e^{\pm 2\pi i k/N}|\bar{k}\rangle_\omega$ and equation (10) that indeed they have the correct eigenvalues, provided the QFT they have is equation (8), instead of our standard notation (different in the negative sign).

(c-1) Based on equation (1)-(6) and equation (15), why does the circuit right below equation (15) have this control NOT from the first qubit of walker space onto the coin space qubit? Based on the implementation, is the first qubit of the walker space most significant qubit or least significant qubit?

(c-2) Based on equation (1)-(6) and equation (15), does the circuit right below equation (15) faithfully implement equation (6) ? And why/why not?

Problem 9 Your Project: 10 pts

Please briefly describe what you have learned in your research project and this topic will evolve in the near future.

Problem 10 Scratch Paper Area 1

Use this sheet if you need extra space. **DO NOT** detach this sheet from the exam.

Problem 11 Scratch Paper Area 2

Use this sheet if you need extra space. **DO NOT** detach this sheet from the exam.