# CS 528: Quantum Computing

Midterm Exam

Date: 10/19/2017

**Instructions:**
You have **80** minutes for this exam. There are 8 problems in this exam and the very last problem (the 8th) is a bonus problem (15 pts). Please work on the problems you have confidence in first. If you finish the exam early, please just leave your exam on the desk and I will collect it. I leave plenty of space on each page for your computation. One extra blank sheet is also attached at the end of this exam if you need more computation space. If your answer is incorrect but you show the computation process, then partial credits will be given. **Please use your time and space wisely**.

**First Name**:

**Last Name**:

**Group ID**:

**Score**: **/85 + /15**

## Problem 1 Orthonormal Basis + Inner Product: 5+5 pts

(a)Please determine if the following is orthonormal basis $\{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix}\}$

(b) Show the result of $\langle 11|\alpha\beta\rangle + \langle 10|\beta\alpha\rangle$ where $|\alpha\rangle = \frac{\sqrt{2}}{\sqrt{11}}|0\rangle + \frac{3}{\sqrt{11}}|1\rangle$ , $|\beta\rangle = \frac{3}{\sqrt{11}}|0\rangle - \frac{\sqrt{2}}{\sqrt{11}}|1\rangle$

## Problem 2 Deutsch Algorithm: 20pts

In the Deutsch algorithm, when we consider $U_f$ as a single-qubit operator $\hat{U}_{f(x)}$ , $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$ is an eigenstate of $\hat{U}_{f(x)}$, whose associated eigenvalue gives us the answer to the Deutsch problem. Suppose we did not prepare $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$ but $|\mathbf{0}\rangle$ instead in the target qubit and we just run the same circuit on that configuration. Please compute and explain what happens at the end of measurement. Furthermore, please conclude the probability that we get the right answer. Please show your computation.

## Problem 3 Gram-Schmidt Orthonormalization:(5+10) pts

Let $b$ be a basis that $b_1 = (1, 0, 1), b_2 = (0, 1, 1)$ and $b_3 = (1, 1, 0)$.
(a) Is $b$ an orthonormal basis? If not, which condition does it violate?

(b) Use the Gram-Schmidt procedure to find the corresponding $(x_1, x_2, x_3)$, and $(y_1, y_2, y_3)$.

## Problem 4 Schroedinger Equation: 10 pts

When we have $|\psi(t)\rangle = U|\psi(0)\rangle$, where $U = e^{\frac{-iHt}{\hbar}}$, then $|\psi(t)\rangle$ is a solution to the Schroedinger equation (Please make sure your explanation contains 1. Schroendinger Equation and 2. the rules for the derivations)

## Problem 5 Simple Computation: 10 pts

We know $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$.
(a) Show $\{|+\rangle, |-\rangle\}$ is an orthonormal basis

(b) Show $|+\rangle\langle+| + |-\rangle\langle-| = |0\rangle\langle 0| + |1\rangle\langle 1|$

## Problem 6 Superdense Coding: 5 + 5 pts

(1) Draw the circuit for superdense coding

(2) Describe and verify how Alice sends the message 10 to Bob using the circuit you drew.

## Problem 7 Hadamard + QPE: 5 + 5 pts

Let $X, Y$ be two $n$-bit strings that $X = x_1x_2...x_n$ and $Y = y_1y_2...y_n$ where $x_i, y_i \in \{0,1\}$. Please prove the following:
(a) When $n = 2$, show $H^{\otimes 2}|X\rangle = \frac{1}{\sqrt{2^2}} \sum_{Y \in \{0,1\}^2} (-1)^{X \cdot Y} |Y\rangle$ where $X \cdot Y = \sum_{i=1}^{2} x_i y_i$

(b) Draw the circuit for quantum phase estimation. Please include both stages where stage 1 is the phase kick back and stage 2 is ($QFT^\dagger$ + measurement).

## Problem 8 Teleportation Gone Wrong!!!: 5+10 pts

In teleportation, Alice and Bob share an entanglement state $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. But unfortunately in this instead the pair of qubits they share now is one of the other Bell states. Let suppose they just follow the standard teleportation protocol to teleport state $|\psi\rangle$ from Alice to Bob using this wrong pair. Please show

(a) Show the state that Bob will receive at the end of the protocol is, up to a phase factor, some Pauli operator (X, Y, or Z) applied to $|\psi\rangle$ . [Hint:Bell states are in this form $(I \otimes \sigma)|\beta_{00}\rangle$ where $\sigma \in \{I, X, Z, XZ\}$.]

(b) Suppose Alice and Bob know they are sharing $\beta_{01} = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle$. Show how they can modify the protocol to make sure teleportation still works for this Bell state they share.

## Problem 9 Scratch Paper Area 1

Use this sheet if you need extra space. **DO NOT** detach this sheet from the exam.