CS 528: Quantum Computation
Problem Set 1

TR: 10:00 - 11:15 am

Out: 09/12/2017 Due: 09/21/2017

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. Please directly hit the point when solving a problem. Cumbersome description might receive fewer credits, even it is correct. If your answer is incorrect but you your logic is on the right track, then partial credits will be given. Please staple your solution and use the space wisely.

First Names:

Group ID:

Score: **/100**

Problem 1 Trace: 15+5pts

(a) Show that for any intergers $m, n \geq 1$, if A is an $n \times m$ matrix and B is an $m \times n$ matrix, then

$$tr(AB) = tr(BA)$$

(b) Show that $tr(ABC) = tr(BCA)$ [simple application of (a)]

Problem 2 Gram-Schmidt Orthonormalization:(5+20) pts

Let b be a basis that $b_1 = (-3, 0, 4), b_2 = (3, -1, 2)$ and $b_3 = (0, 1, -1)$.
(a) Is b an orthonormal basis? If not, which condition does it violate?

(b) Use the Gram-Schmidt procedure to find the corresponding (x_1, x_2, x_3), and (y_1, y_2, y_3).

Problem 3 Schroedinger Equation: 10 + 10 pts

(a) When we have $|\psi(t)\rangle = U|\psi(0)\rangle$, where $U = e^{\frac{-iHt}{\hbar}}$, then $|\psi(t)\rangle$ is a solution to the Schroedinger equation

(b) We learned about the exponential map, and lots of times we need to look at things from different basis. Given the idea that bruteforcely computing an exponetial function of a Hamiltonian H might be **hard and resource consuming**, i.e. computing $U = e^{\frac{-iHt}{\hbar}}$ for various t, what is the advantage to change basis from regular computational basis to the eigenbasis of H when computing? [Hint: if in eignebasis and full rank, what happens to the computation of exponentiating a matrix?]

Problem 4 Simple Computation: 25 pts

Suppose we are given:

$$T = |1\rangle\langle 1| \otimes (|+\rangle\langle +| - |-\rangle\langle -|) + |0\rangle\langle 0| \otimes (|+\rangle\langle +| + |-\rangle\langle -|)$$

where $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$.
(a) Show $\{|+\rangle, |-\rangle\}$ is an orthonormal basis

(b) Show that T is CNOT operator

Problem 5 Entanglement: 5+ 5 pts

(a) Please read the EPR paradox on Wikipedia

Yes, I did read it
No, I did not read at all

(b) Draw the circuit for generating one of the Bell states (your choice of which) and please verify your circuit