CS 528: Quantum Computation
Problem Set 2

TR: 10:00 - 11:15 am

Out: 10/11/2017 Due: 10/24/2017

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. Please directly hit the point when solving a problem. Cumbersome description might receive fewer credits, even it is correct. If your answer is incorrect but you your logic is on the right track, then partial credits will be given. Please staple your solution and use the space wisely. The bonus problem is about the blochsphere and is given in a separate file.

First Names:

Group ID:

Score: **/110 + bonus /(4*5 = 20)**

Problem 1 Teleportation: 15pts

In class we talked about teleportation. However, it is only for 1-qubit. Let say you are the leading scientist in your research team and it is very much needed that you can come up with a protocol to teleport 2-qubits. How would you modify the 1-qubit protocol to make it happen? If it is not possible for now, what constraint can you impose on your state (the state to be transported) such that simple adaption of the original 1-qubit teleportation protocol would work? Verify your idea.

Problem 2 Superdense Coding: 5 + 15 pts

(1) Draw the circuit for superdense coding

(2) Describe and verify how Alice sends the message 11 to Bob using the circuit you drew.

Problem 3 Hadamard in the Deutsch-Jozsa Algorithm: 5+10pts

Let X, Y be two n-bit strings that $X = x_1x_2...x_n$ and $Y = y_1y_2...y_n$ where $x_i, y_i \in \{0,1\}$. Please prove the following:

(a) When $n = 3$, show $H^{\otimes 3}|X\rangle = \frac{1}{\sqrt{2^3}} \sum_{Y\in\{0,1\}^3} (-1)^{X\cdot Y}|Y\rangle$ where $X \cdot Y = \sum_{i=1}^{3} x_i y_i$

(b) $H^{\otimes n}|X\rangle = \frac{1}{\sqrt{2^n}} \sum_{Y\in\{0,1\}^n} (-1)^{X\cdot Y}|Y\rangle$ where $X \cdot Y = \sum_{i=1}^{n} x_i y_i$

Problem 4 Deutsch Algorithm: 20 pts

In the Deutsch algorithm, when we consider U_f as a single-qubit operator $\hat{U}_{f(x)}$, $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$ is an eigenstate of $\hat{U}_{f(x)}$, whose associated eigenvalue gives us the answer to the Deutsch problem. Suppose we did not prepare $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$ but $|1\rangle$ instead in the target qubit and we just run the same circuit on that configuration. Please compute and explain what happens at the end of measurement. Furthermore, please conclude the probability that we get the right answer. (Ex. 6.3.1 in the required textbook)

Problem 5 Eigenstates for SWAP: 8 + 4*3 pts

Recall the two-qubit swap operator SWAP satisfying $SWAP|a\rangle|b\rangle = |b\rangle|a\rangle$ for all $a, b \in \{0, 1\}$. Please show
(a) circuit implementation of SWAP gate using CNOT gates

(b) the four Bell states are eigenvectors of SWAP and their associated eigenvalues

Problem 6 Quantum Phase Estimation (QPE): 20 pts

(a) Draw the circuit for quantum phase estimation. Please include both stages where stage 1 is the phase kick back and stage 2 is ($QFT^{\dagger}$ + measurement).

(b) The core idea of QPE can be simply described in equation 7.1.17 (for stage 2) and equation 7.1.18 (for stage 1) in the textbook (Kaye, Laflamme, and Mosca). Please verify equation 7.1.18.

Problem 7 Practice Problems (no need to turn in)

Let $\{|0\rangle, |1\rangle, |2\rangle, |3\rangle\}$ be the standard orthonormal basis in $\mathbb{C}^4$. The cyclic shift operator S permutes the basis vectors as follows: $|0\rangle \to |1\rangle, |1\rangle \to |3\rangle, |3\rangle \to |2\rangle, |2\rangle \to |0\rangle$. Let $|\psi\rangle := \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle + \alpha_3|3\rangle$. Please compute the following:
(a) (9 pts) Determine the matrix representation for S^3 and $S^\dagger$.
(b) (3 pts) Compute $S^2|\psi\rangle$.
(c) (3pts) Computer $\langle\psi|(S|\psi\rangle)$.

Problem 13.2: (1)(2)(3) Circuit verification on page 85-86 in SF Note

Problem IV: (a)(b) in the Midterm section on page 122 in the SF Note

Problem 3: Euclidean distance on page 26 in the RW Note