MAT 115: Exam 1

Section: TR 4-5:50 pm

Date: 10/05/2017

Instructions:
You have **110** minutes for this exam. There are 10 + 1 problems. The total score is **100** plus extra 10 points from a bonus problem. Problem 12 is the set of algebraic rules for your reference. Please work on those in which you have confidence. If you finish the exam early, please just leave your exam on the desk and I will collect it. I leave plenty of space on each page for your computation. One extra blank sheet is also attached at the end of this exam if you need more computation space. If your answer is incorrect but you show the computation process, then partial credits will be given. Please use the blank space in the exam and your time wisely.

First Name:

Last Name:

Score: /**100** + /**10**

Problem 1 Boolean + Base Change (2+2+3+3 pts)

Given a function $f : \{0,1\}^5 \rightarrow \{0,1\}$, please answer the following :
(a) Please show 2 input instances from in the domain.

(b) How many possible input instances are there in the domain?

(c) What is the number of possible boolean functions f?

(d)Convert the following number: 1F0C (from base 16 to base 13)
(hint: in base 16, A=10, B=11, ..., F=15) [must show steps, cannot just use calculator and throw out a number]

Problem 2 Logic: Quantifiers (3+3+4 pts)

Is the following statement true or false? Explain why. Your reasoning path counts, even it leads to a wrong conclusion.
(a) $\forall x \in D, (P(x) \wedge Q(x))$ is the same as $(\forall x \in D, P(x)) \wedge (\forall x \in D, Q(x))$

(b) $\exists x \in D, (P(x) \wedge Q(x))$ is the same as $(\exists x \in D, P(X)) \wedge (\exists x \in D, Q(x))$

(c) $\forall x P(x) \vee \forall x Q(x)$ and $\forall x(P(x) \vee Q(x))$ are equivalent

Problem 3 Proof: 10pts

Please show that for any integer m and n, $m^2 - n^2$ is even **if only if** $m - n$ is even.
(a) Given any integer m and n, if $m^2 - n^2$ is even, then $m - n$ is even

(b) Given any integer m and n, if $m - n$ is even, then $m^2 - n^2$ is even

Problem 4 Nested Quantifier: 2*5pts

Let $Q(x)$ be the statement $x + y = x - y$, given $x, y, z \in \mathbb{Z}$. what are the truth value of the following expressions?

(a) $\exists x \exists y\ Q(x, y)$

(b) $\exists y \forall x\ Q(x, y)$

(c) $\forall x \forall y\ Q(x, y)$

(d) $\forall x \forall y \exists z\ (z = (x + y)/2)$

(e) same as (d) but now $x, y, z \in \mathbb{R}$

Problem 5 Truth Table (5 pts)

Make a truth table for $f = (q \vee (r \wedge \sim q)) \wedge \sim (\sim p \vee \sim r)$.

p	q	r	**f**
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Problem 6 Proof: Algebraic Rules: 10pts

Is the function $(r \vee p) \wedge (\sim r \vee (p \wedge q)) \wedge (r \vee q)$ equla to the function $p \wedge q$

Problem 7 Predicate Logic:10pts

$D = \{1, 3, 4, 5, 9, 121, 169, 196, 225, 289\}$, $S(x) = (\sqrt{x} \in \mathbb{Z} \wedge \sqrt{x} + 2 \notin \mathbb{P})$ where $\mathbb{P}$ is the set of prime numbers. Let truth set $T = \{x \in D | S(x)\}$. Please show the elements inside the set T [the symbol $\notin$ means NOT IN].

Problem 8 Floor Ceiling Functions:5 +5 : 10pts

Please compute the following:
(a) $\lceil(\lfloor 3.45\rfloor * \lceil -8.7\rceil + 7.4)\rceil * 2$

(b) $(\lfloor -2.4\rfloor * \lceil 4.6\rceil) + \lceil(2.5 * 6)\rceil$

Problem 9 Simple Induction: 15pts (2+ 3 + 10)

Please use induction proof method to show that $\sum_{i=1}^{n} i^2 + 3i = \frac{n(n+1)(n+5)}{3}$

Base case:

Hypothesis:

Induction:

Problem 10 Euclidean Algorithm: (5 + 5)pts

Using the Euclidean algorithm on $m = 326$ and $n = 86$
(a) Find gcd(m, n)

(b) Find integers A and B such that $Am + Bn = gcd(m, n)$

Problem 11 Bonus: Circuit Complexity (5+5) pts

In class we talk about half adder and full adder for addtion of two binary strings. Let say we have two binary strings of length n: $S1 = a_{n-1}a_{n-2} \cdot a_2a_1a_0$ and $S2 = b_{n-1}b_{n-2} \cdot b_2b_1b_0$

(1) We know the biggest $S1 = 1111\cdots 1$ (a bit string with n 1s). What is its value in decimal system? (Please show the final result, do not write $2^0 + 2^1 + \cdots$)

(2) Let say if each elementary gate (AND, NOT, OR, XOR) takes T seconds to operate. Then how much time does it take to add $S1$ and $S2$? [Hint: nT is wrong... remember a half adder has two elementary gates operated in parallel (AND and XOR gates) and a full adder has 3 stages (one half adder, one half adder and then OR gate). In parallel that means gates can run at the same time, therefore an half adder only takes T seconds to operate. If you draw the circuit and count, you might find it easy to figure the complexity of the addition operation on 2 n-bit binary strings]

Problem 12 Algebraic Rules Sheet

Theorem 2 (Algebraic rules for Boolean functions) *Each rule states that two different-looking Boolean functions are equal. That is, they look different but have the same table.*

Associative Rules:	$(p \wedge q) \wedge r = p \wedge (q \wedge r)$	$(p \vee q) \vee r = p \vee (q \vee r)$
Distributive Rules:	$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$
Idempotent Rules:	$p \wedge p = p$	$p \vee p = p$
Double Negation:	$\sim\sim p = p$	
DeMorgan's Rules:	$\sim(p \wedge q) = \sim p \vee \sim q$	$\sim(p \vee q) = \sim p \wedge \sim q$
Commutative Rules:	$p \wedge q = q \wedge p$	$p \vee q = q \vee p$
Absorption Rules:	$p \vee (p \wedge q) = p$	$p \wedge (p \vee q) = p$
Bound Rules:	$p \wedge 0 = 0 \qquad p \wedge 1 = p$	$p \vee 1 = 1 \qquad p \vee 0 = p$
Negation Rules:	$p \wedge (\sim p) = 0$	$p \vee (\sim p) = 1$

Problem 13 Scratch Paper

Do not detach the paper.

Problem 14 Scratch Paper

Do not detach the paper.