MAT 115: Exam II

Section: TR 4-5:50 pm

Date: 11/14/2017

Instructions:
You have **110** minutes for this exam. There are 9 + 1 problems. The total score is **100** plus extra 10 points from a bonus problem. Problem 11 is the set of set algebraic rules for your reference. Please work on those in which you have confidence. If you finish the exam early, please just leave your exam on the desk and I will collect it. I leave plenty of space on each page for your computation. One extra blank sheet is also attached at the end of this exam if you need more computation space. If your answer is incorrect but you show the computation process, then partial credits will be given. Please use the blank space in the exam and your time wisely.

First Name:

Last Name:

Score: **/100 +** **/10**

Problem 1 Use of Subtraction Rule: 5 + 5 pts

Prove each of the following identities from the basic algebraic rules for sets. You may want to use the fact that $D - E = D \cap \sim E$ [Here the $\sim$ means the complement].
(a) If A and B are subsets of U, then $A = (A \cup B) - (\sim A \cap B)$.

(b) If A, B, and C are subsets of U, then $(A - B) - C = A - (C \cup B)$

Problem 2 Lexical Order: 5 + 5 pts

Let $A = \{1, 2\}, B = \{u, v\}$, and $C = \{m, n\}$. Take the linear order on A to be numeric and the linear orders on B and C to be alphabetic. List the elements in each of the following sets in lexicographic order.
(a) $(A \times B) \times C$

(b) $A \times C \times B$

Problem 3 Set Proof: 10pts

Prove or give a counterexample. Use algebraic rules to prove. For the counterexample, use a Venn diagram or use set specialization.
(a) If A, B, and C are subsets of U, then $(A - C) \cap (B - C) \cap (A - B) = \emptyset$.

(b) If A and B are subsets of U and if $A \subseteq B$, then $A \cap (U - B) = \emptyset$.

Problem 4 Power Set: 5 + 5pts

Compare the following pairs of sets. If they are equal, please proof. If not, please show an counter exampl. Here the $\mathbb{P}(A)$ means the power set of A.
(a) $\mathbb{P}(A \cup B)$ and $\mathbb{P}(A) \cup \mathbb{P}(B)$

(b) $\mathbb{P}(A \times B)$ and $(\mathbb{P}(A) \times \mathbb{P}(B)$

Problem 5 Permutation In Various Forms: 6*2 = 12 pts

This exercise lets you check your understanding of cycle form. A permutation is given in one-line, two-line or cycle form. Give its inverse in all three forms.
(a) (1,7,8) (2,3) (5,4) (6).

(b) (5,2,3,4,1), which is in one line form.

Problem 6 Relation & Partition : 10 (6 + 3 + 1) pts

Let $A = \{1, 2, 3\}$ and $R = \{(1, 1), (2, 2), (3, 3), (2, 3), (3, 2)\}$.
(a) Is R an equivalence relation? (need to verify those three properties)

(b) What are the equivalent classes (partitions) of A

(c) $A = \{1, 2, 3, 4, 5\}$ and it can be partitioned by R_2 such that the partitions are $P_1 = \{1, 5\}, P_2 = \{2, 4\}$ and $P_3 = \{3\}$. Please write out the relation R_2.

Problem 7 Equivalence Relation: : 15pts

Let the set be $S = Z \times Z^*$, where Z is the set of all integers except 0. Let R be an binary relation that acts on S. We write $(a, b)R(c, d)$, i.e., the pair $((a, b), (c, d)) \in R$, if and only if $ad = bc$ Please show that R is an equivalence relation.
(a) Reflexive:

(b) Symmetric:

(c) Transitive:

Problem 8 Equivalence Relation: 10 pts

Define an equivalence relation R on the positive integers $A = \{2, 3, 4, ..., 29\}$ by mRn if the largest prime divisor of m is the same as the largest prime divisor of n. Please write out all the equivalence classes of R

Problem 9 Proof: Combination Formula 13: 5 + 3 + 5 pts

We talked about the combination. $C(n,k)$ means how many ways one can pick k distinct items out of n distinct items (grab k items at one time out of n items). It is know that $C(n,k) = \frac{n!}{(n-k)!k!}$ where $T! = 1 \times 2 \times 3 \times \cdots \times T$. Please show

(a) $\frac{n!}{(n-k)!k!} = (n \times (n-1) \times \cdots \times (n-k+1))/k!$

(b) What is the value of $C(100,2)$?

(c) Please show that $C(n-2,k-2)+C(n-2,k-1)+C(n-2,k-1)+C(n-2,k) = C(n,k)$ (provided you know $C(n-1,k-1)+C(n-1,k) = C(n,k)$ for all positive integers n,k and $k < n$).

Problem 10 Bonus: Types of Functions: 10 (3 + 3 + 4)pts

Let A and B be finite sets and $f : A \to B$. Prove the following claims. Some are practically restatements of the definitions, some require a few steps.
(a) If f is an injection, then $|A| \leq |B|$.

(b) If f is a surjection, then $|A| \geq |B|$.

(c) Given A=$\{1, 2, 3, 4\}$ and B=$\{\alpha, \beta, \gamma, \delta\}$. We have the following: $f(1) = \alpha, f(2) = \beta, f(3) = \gamma, f(4) = \gamma$. What type is the function f?

Problem 11 Set Algebraic Rules Sheet

Associative:	$(P \cap Q) \cap R = P \cap (Q \cap R)$	$(P \cup Q) \cup R = P \cup (Q \cup R)$
Distributive:	$P \cap (Q \cup R) = (P \cap Q) \cup (P \cap R)$	$P \cup (Q \cap R) = (P \cup Q) \cap (P \cup R)$
Idempotent:	$P \cap P = P$	$P \cup P = P$
Double Negation:	$\sim\sim P = P$	
DeMorgan:	$\sim(P \cap Q) = \sim P \cup \sim Q$	$\sim(P \cup Q) = \sim P \cap \sim Q$
Absorption:	$P \cup (P \cap Q) = P$	$P \cap (P \cup Q) = P$
Commutative:	$P \cap Q = Q \cap P$	$P \cup Q = Q \cup P$

Problem 12 Scratch Paper

Do not detach the paper.

Problem 13 Scratch Paper

Do not detach the paper.