MAT 115: Exam III

Section: TR 4-5:50 pm

Date: 12/07/2017

Instructions:
You have **110** minutes for this exam. The total score is **100** plus extra 10 points from a bonus problem. Please work on those in which you have confidence. If you finish the exam early, please just leave your exam on the desk and I will collect it. I leave plenty of space on each page for your computation. One extra blank sheet is also attached at the end of this exam if you need more computation space. If your answer is incorrect but you show the computation process, then partial credits will be given. Please use the blank space in the exam and your time wisely.

First Name:

Last Name:

Score: **/100 +** **/10**

Problem 1 Permutation + Combinatorics: (20 pts)

Please explain how many ways to put balls into bins based on the condition:
(a) 3 identical bins and 3 identical balls

(b) 3 different bins (A, B, C) and 3 identical balls

(c) 3 identical bins and 3 different balls (Red, White, Green)

(d) 3 different bin (A, B, C) and 3 different balls (Red, White, Green)

Problem 2 Permutation: With Repetition ($3 \times 5 = 15$ pts)

We work with the ordinary alphabet of 26-letters. Please solve the following:
(a) Define a 5-letter word to be any list of 5 letters that contains *at least* one of the vowels A, E, I, O and U. How many 5-letter words are there?

(b) How many 5-letter words with exactly 1 vowel that if A is chosen, so must B (that is if A appears (might be more than 1 time), B must appear at least once).

(c) How many 5-letter words with exactly 2 vowels such that if S appears, so must T (1S has 1 T). S and T must be adjacent to each other

Problem 3 Permutation: ($5 \times 3 = 15$ pts)

We are interested in forming 3 letter words using the letters in LITTLETENNESSEE.
(a) How many words can be made with no repeated letters?

(b) How many words can be made with unlimited repetition allowed?

(c) How many words can be made if repeats are allowed but no letter can be used more than it appears in LITTLETENNESSEE?

Problem 4 Probability + Counting $((3\times\ 3)\times 3) = 27$ pts)

An urn contains nine labeled balls, labels 1,2, $\cdots$, 9.
(a) Two balls are **drawn together**. What is the sample space? What is the probability that the sum of the labels on the balls is odd? What is the probability that the sum of the labels on the balls is 7?

(b)Two balls are drawn one after the other **without replacement** and the order matters. What is the sample space? What is the probability that the sum of the labels on the balls is odd? What is the probability that the sum of the labels on the balls is 11?

(c) Two balls are drawn one after the other **with replacement and the order matters**. What is the sample space? What is the probability that the sum of the labels on the balls is even? What is the probability that the sum of the labels on the balls is 8?

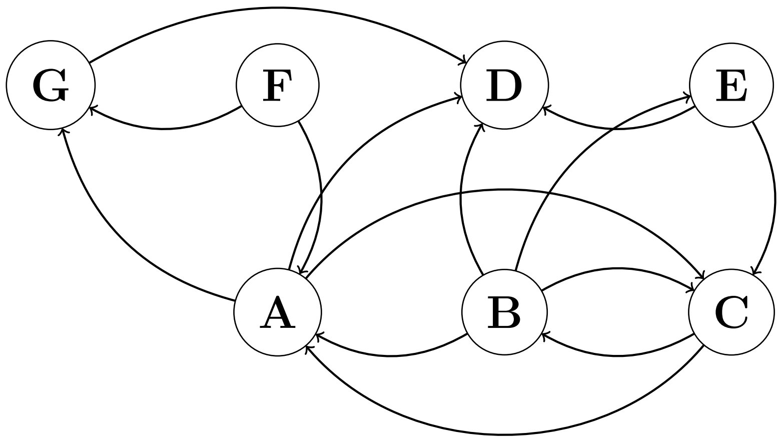
Problem 5 Probability + Counting ((2+3)*2 = 10 pts)

(a) We have five girls and three boys sit in a row. Let us suppose we cannot distinguish the girls and we cannot distinguish the boys. How many ways they can sit? And what is the probability that each of the boy sits between two girls?

(b)We have five girls (A, B, C, D, E) and three boys (F, G, H) sit in a row. How many ways they can sit? And what is the probability that each of the boy sits between two girls and A must sit at the first position?

Problem 6 Graph: 3 + (1+2+2) + 5 pts

Suppose you are given the following **directed** graph $G = (V, E)$.



(a) Find Cycle 1 contains 3 distinct vertices.

(b) Is is possible to find a cycle of more than 3 distinct vertices? If not possible, what absorbing/radiating nodes are causing the problem? And what nodes are affected by those ?

(c) Please think of a real life problem to be presented in a graph. Please exactly define (1) nodes (2) edges (3) your objective function.

Problem 7 Bonus: Stirling Number: 3 + 7 pts

For $n > k > 0$, the Stirling number of the 2nd kind is $S(n,k) = S(n-1,k-1) + k \times S(n-1,k)$. A way to interpret it is how many ways to put n distinct objects into k identical bins while none of the bins should be empty.
(a) What is the value of $S(5,3)$?

(b) Prove that $S(n,n-1) = C(n,2)$

Problem 8 Scratch Paper

Do not detach the paper.