MAT 115: Finite Math for Computer Science
Problem Set 2

Due: 10/03/2017

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Group ID:

Score: **/120 + 10 (bonus)**

Problem 1 Proof: Direct + Modulo Function: 10pts

Prove or give a counterexample: The product of any four consecutive integers is equal to 0 (mod 8). [hint: use modulo 4 to represent those four integers]

Problem 2 Propositional Logic: 5pts

Is the statement form $((\sim p \wedge q) \wedge (q \vee \sim r)) \wedge \sim (q \wedge r)$ a tautology, contradiction or neither? Please use algebraic rules.

Problem 3 Proof:10pts

For all distinct positive integers m and n, both m and n are perfect squares if and only if $m + 2m^{1/2}n^{1/2} + n$ is a perfect square. Prove if correct, otherwise show counter examples
(a) Show positive perfect square integers m and $n \to (m+2m^{1/2}n^{1/2}+n)$ is a perfect square

(b) A perfect square $(m + 2m^{1/2}n^{1/2} + n) \to m, n$ are two postive distinct perfect squares
[hint: contrapositive will make your life easier ...]

Problem 4 Propositional Logic: 5pts

Is $(p \wedge \sim q) \wedge (\sim p \vee q) \wedge r$ a tautology, contradiction or neither? Please use algebraic rules.

Problem 5 Predicate Logic: 4+3+3:10pts

Start with the statement, "$\forall n \in \mathbb{N}$, if n^2+1 is even, then n is odd". Form the contrapositive, converse, and inverse of the statement. Which statements are true? [Hint: Let p be defined as $\forall n \in \mathbb{N}$, if $n^2 + 1$ is even. Let q be defined as n is odd. The statement is $p \to q$.]

Problem 6 Predicate Logic:10pts

$D = \{1, 3, 4, 5, 9, 121, 169, 196, 225, 289\}$, $S(x) = (\sqrt{x} \in \mathbb{Z} \wedge \sqrt{x+2} + 2 \notin \mathbb{P})$ where $\mathbb{P}$ is the set of prime numbers. Let truth set $T = \{x \in D | S(x)\}$. Please show the elements inside the set T.

Problem 7 Proof:10pts

Prove or disprove the following:
(a) The difference of two integers is even **if and only if** at least one of them is even

(b) The product of two integers is even **if and only if** at least one of them is even

Problem 8 Floor Ceiling Functions:5 +5 : 10pts

Please compute the following:
(a) $\lceil(\lfloor 3.85\rfloor * \lceil -6.3\rceil + 2.5)\rceil * 3$

(b) $(\lfloor -3.4\rfloor * \lceil 5.3\rceil) + \lceil(2.5 * 3.3)\rceil$

Problem 9 Predicate Logic: 5 + 5: 10pts

(a) Why the statemet $\forall x \in D, (P(x) \vee Q(x))$ is not the same as $(\forall x \in D, P(x)) \vee (\forall x \in D, Q(x))$? Given an example.

(b) Why the statemet $\exists x \in D, (P(x) \wedge Q(x))$ is not the same as $(\exists x \in D, P(x)) \wedge (\exists x \in D, Q(x))$? Given an example.

Problem 10 Nested Quantifier: Q.26, Q28 on P.67: 3*5pts

Let $Q(x)$ be the statement $x + y = x - y$, given $x, y, z \in \mathbb{Z}$. what are the truth value of the following expressions? And why?

(a) $\exists x \exists y\, Q(x, y)$

(b) $\exists y \forall x\, Q(x, y)$

(c) $\forall x \forall y\, Q(x, y)$

(d) $\forall x \forall y \exists z\, (z = (x + y)/2)$

(e) same as (d) but now $x, y, z \in \mathbb{R}$

Problem 11 Simple Induction: 15pts (2+ 3 + 10)

Please use induction proof method to show that $\sum_{i=1}^{n} i^2 + i + 1 = \frac{n((n+1)(n+2)+3)}{3}$

Base case:

Hypothesis:

Induction:

Problem 12 Euclidean Algorithm: 10pts

Using the Euclidean algorithm, find A and B such that $Am+Bn = gcd(m, n)$ where $m = 163$ and $n = 86$.

Problem 13 Practice Problems

For practice only. You do not have to turn in the solution.
Unit Lo: 1.19, 2.13, 2.19.11
Unit NT: 1.6(a)(b), 1.14, 1.25(a), 1.28