MAT 115: Finite Math for Computer Science
Problem Set 4

Due: Noon 11/13/2017

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Group ID:

Score: **/125**

Problem 1 Equivalence Relation: 15pts

Let the binary relation R act on the integer set $\mathbb{Z}$. Let d, k be two positive integers and R be defined that $(x, y) \in R$ if $d|(x^k - y^k)$ where $x, y \in \mathbb{Z}$. Show R is an equivalence relation.
(a) Reflexive:

(b) Symmetric:

(c) Transitive:

Problem 2 Relation & Partition : 10 + 5 + 5 pts

Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 4), (4, 1), (2, 3), (3, 2)\}$.
(a) Is R an equivalence relation? (need to verify those three properties)

(b) What are the equivalent classes (partitions) of A

(c) $A = \{1, 2, 3, 4, 5, 6\}$ and it can be partitioned by R_2 such that the partitions are $P_1 = \{1, 5\}, P_2 = \{2, 4, 6\}$ and $P_3 = \{3\}$. Please write out the relation R_2.

Problem 3 Equivalence Relation: : 15pts

Let the set be $S = Z \times Z^*$, where Z is the set of all integers except 0. Let R be an binary relation that acts on S. We write $(a, b)R(c, d)$, i.e., the pair $((a, b), (c, d)) \in R$, if and only if $ad = bc$ Please show that R is an equivalence relation.

(a) Reflexive:

(b) Symmetric:

(c) Transitive:

Problem 4 Equivalence Relation: 15 (10 + 5) pts

Define an equivalence relation R on the positive integers $A = \{2, 3, 4, ..., 20\}$ by mRn if the largest prime divisor of m is the same as the largest prime divisor of n.
(a) Write out all the pairs in R

(b) Based on (a), what is the number of equivalence classes of R?

Problem 5 Pigeonhole: 10 + 5 pts

Given $N \in \mathbb{N}$, find minimal t such that for every list $A = (a_1, ..., a_t)$ of t distinct integers, there exist $i \neq j$ and $k \neq m$ such that $a_i + a_j = a_k + a_m$ (mod N) and $\{i, j\} \neq \{k, m\}$.
(a) Show how large t should be at least such that the above situatin occurs

(b) If $N = 21$, what is the miminal t that makes this happen?

Problem 6 Proof: Combinatorial 15: 5 + 10 pts

We talked about the combination. $C(n,k)$ means how many ways one can pick k distinct items out of n distinct items (grab k items at one time out of n items). It is know that $C(n,k) = \frac{n!}{(n-k)!k!}$ where $T! = 1 \times 2 \times 3 \times \cdots \times T$ for any positive integer T. Please show

(a) $\frac{n!}{(n-k)!k!} = (n \times (n-1) \times \cdots \times (n-k+1))/k!$

(b) Please show that $C(n-1,k-1) + C(n-1,k) = C(n,k)$ [For instance, you can verify that $C(4,2) + C(4,3) = C(5,3)$].

Problem 7 Pigeon Holes : 10pts

There are N students in a class. Their exam scores ranged between 27 and 94. All possible scores were achieved by at least one student except for the scores 31, 43, and 55 (none of the students got these scores). What is the smallest value of N that guarantees that at least three students achieved the same score?

Problem 8 Types of Functions: 20 (5 + 3 + 5 + 3 + 4)pts

Let A and B be finite sets and $f : A \to B$. Prove the following claims. Some are practically restatements of the definitions, some require a few steps.
(a) If f is an injection, then $|A| \leq |B|$.

(a-1) Show an example that $|A| \leq |B|$, but f is not an injection.

(b) If f is a surjection, then $|A| \geq |B|$.

(b-1) Show an example that $|A| \geq |B|$, but f is not an surjeciton.

(c) If f is a bijection, then $|A| = |B|$.

Problem 9 No need to turn in but candidates for exam problems

1. Q2. on Fn-41
2. Q4 on Fn-41
3. Q6 on Fn-42
4. Q5 on EO-34
5. Q6, Q8 on EO-(34,35)