MAT 115: Finite Math for Computer Science
Problem Set 6

Out: 11/28/2017 Due: 12/05/2017

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Group ID:

Score: **/90+10**

Problem 1 Probability + Counting ($3\times$ 3 $\times$ $3 = 27$ pts)

An urn contains eleven labeled balls, labels 1,2, $\cdots$, 11.
(a) Two balls are drawn together. What is the sample space? What is the probability that the sum of the labels on the balls is odd? What is the probability that the sum of the labels on the balls is 7?

(b)Two balls are drawn one after the other without replacement and the order matters. What is the sample space? What is the probability that the sum of the labels on the balls is (odd and the first ball number must be greater than the 2nd ball number)? What is the probability that the sum of the labels on the balls is 9?

(c) Two balls are drawn one after the other with replacement and the order matters. What is the sample space? What is the probability that the sum of the labels on the balls is even? What is the probability that the sum of the labels on the balls is 10?

Problem 2 Stirling Number: (5+5) + 5 + 5 + 5 pts

For $n > k > 0$, the Stirling number of the 2nd kind is $S(n, k) = S(n-1, k-1) + k \times S(n-1, k)$. A way to interpret it is how many ways to put n distinct objects into k identical bins while none of the bins should be empty.
(a) What is the value of $S(5, 2)$? What is the value of $C(5, 2)$?

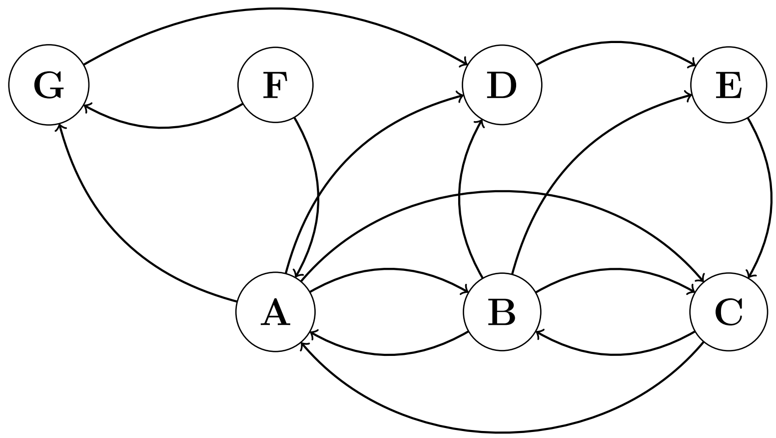
(b) Prove that $S(n, n-1) = C(n, 2)$

(c) Show $S(n, 2) = 2^{n-1} - 1$ (Hint: recursive calls add up the exponents)

(d) What is Sitrling number of the first kind? What are the applications for Stirling number of the 2nd kind?

Problem 3 Graph: Definition and Cycles: $3 \times 3 + 5 + 10$ pts

Suppose you are given the following **directed** graph $G = (V, E)$.



(a) Find 3 cycles within the graph. Cycle 1 contains 4 distinct vertices; cycle 2 contains 5 distinct vertices; Cycle 3 contains 6 distinct vertices.

(b) Is is possible to find a cycle of 7 distinct vertices? And why?

(c) Let say we remove node F and G from this graph and each edge is associated with a weight. What algorithm would you use to find the Hamiltonian cycle? And how does it work?

Problem 4 Graph: Edges: 7 + 7 pts

For a graph $G = (V, E)$, let $d(v)$ be the degree of the vertices $v \in V$. Prove that
(a) $\sum_{v \in V} d(v) = 2|E|$, an even number.

(b) Conclude that number of vertices v for which $d(v)$ is odd is even.

Problem 5 Bonus: Computation Skill + Pigeon Hole: 4 + 6 pts

In the two-sum property problem, we are given $S = \{1, 2, \cdots, 16\}$ and we are asked to find the the smallest number k such that a subset, T, of S contains k distinct elements from S and this subset T must have the two-sum property. In the computation, we know we are computing the number of inputs (possible subsets of T) and the number of outputs (possible sums for those subsets). We want to establish the fact that the cardinality of input is greater than the cardinality of output will make the two-sum property possible for T. Please explain why

(a) The number of possible subsets as input is $2^k - 2$

(b) Please show the steps that the number of possible sums is $16(k-1) - \frac{(k-1)(k-2)}{2}$