MAT 115: Exam 1

Section: MWF 9:20-10:30 pm

Date: 09/28/2018

Instructions:
You have **70** minutes for this exam. The total score is **90** plus extra 10 points from a bonus problem. Problem 12 is the set of algebraic rules for your reference. Please work on those in which you have confidence. If you finish the exam early, please just leave your exam on the desk and I will collect it. I leave plenty of space on each page for your computation. One extra blank sheet is also attached at the end of this exam if you need more computation space. If your answer is incorrect but you show the computation process, then partial credits will be given. Please use the blank space in the exam and your time wisely.

First Name:

Last Name:

Score: **/90 +** **/10**

Problem 1 Truth Table (10 pts)

Make a truth table for $f = ((\sim (p \wedge q) \wedge (\sim p \vee r))) \wedge \sim (q \vee \sim r \vee p)$.

p	q	r	**f**
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Problem 2 Boolean + Base Change (2+2+3+3 pts)

Given a function $f : \{0,1\}^4 \to \{0,1\}$, please answer the following :
(a) Please show 2 input instances from in the domain.

(b) How many possible input instances are there in the domain?

(c) What is the number of possible boolean functions f?

(d)Convert the following number: 1A0B (from base 16 to base 11)
(hint: in base 16, A=10, B=11, ..., F=15) [**must show steps**, cannot just use calculator and throw out a number]

Problem 3 Proof: 10pts

Please show that for any integer m and n, $m^3 - n^3$ is even **if only if** $m - n$ is even.
[hint: $m^3 - n^3 = (m - n)(m^2 + mn + n^2)$]
(a) Given any integer m and n, if $m^3 - n^3$ is even, then $m - n$ is even

(b) Given any integer m and n, if $m - n$ is even, then $m^3 - n^3$ is even

Problem 4 Nested Quantifier: 2*5pts

Let $Q(x)$ be the statement $x + y = x - y$, given $x, y, z \in \mathbb{Z}$. what are the truth value of the following expressions?

(a) $\exists x \exists y\ Q(x, y)$

(b) $\exists y \forall x\ Q(x, y)$

(c) $\forall x \forall y\ Q(x, y)$

(d) $\forall x \forall y \exists z\ (z = (x + y)/2)$

(e) same as (d) but now $x, y, z \in \mathbb{R}$

Problem 5 Propositional Logic: 10pts

Is the statement form $((\sim p \wedge q) \wedge (q \vee r)) \wedge \sim q \wedge r$ a tautology, contradiction or neither? Please use algebraic rules.

Problem 6 Proof: Algebraic Rules: 10pts

Is the function $(r \vee p) \wedge (\sim r \vee (p \wedge q)) \wedge (r \vee q)$ equla to the function $p \wedge q$

Problem 7 Predicate Logic:10pts

$D = \{1, 3, 4, 5, 9, 121, 169, 196, 225, 289\}$, $S(x) = (\sqrt{x} \in \mathbb{Z} \wedge \sqrt{x} + 2 \notin \mathbb{P})$ where $\mathbb{P}$ is the set of prime numbers. Let truth set $T = \{x \in D | S(x)\}$. Please show the elements inside the set T [the symbol $\notin$ means NOT IN].

Problem 8 Floor Ceiling Functions:5 +5 : 10pts

Please compute the following:
(a) $\lceil(\lfloor 3.45 \rfloor * \lceil -8.7 \rceil + 7.4)\rceil * 2$

(b) $(\lfloor -2.4 \rfloor * \lceil 4.6 \rceil) + \lceil (2.5 * 6) \rceil$

Problem 9 Simple Induction: 10pts (2+ 3 + 5)

Please use induction proof method to show that $\sum_{i=1}^{n} i^2 + 3i = \frac{n(n+1)(n+5)}{3}$

Base case:

Hypothesis:

Induction:

Problem 10 Bonus: 10pts

We learned about half adder (HA) and full adder (FA) in class. Let say if we are to add **three** binary numbers, p_1, p_2 and p_3 where p_1 has $k-1$ bits, p_2 has k bits while p_3 has $k+1$ bits.
(1) Draw your cicuit for solving this problem.

(2)Please explain what the gate complexity and the depth of the circuit are if you build the circuit based on full adder and let us assume each gate (AND, OR, NOT, XOR) takes T time to run.

Problem 11 Algebraic Rules Sheet

Theorem 2 (Algebraic rules for Boolean functions) *Each rule states that two different-looking Boolean functions are equal. That is, they look different but have the same table.*

Associative Rules:	$(p \wedge q) \wedge r = p \wedge (q \wedge r)$	$(p \vee q) \vee r = p \vee (q \vee r)$
Distributive Rules:	$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$
Idempotent Rules:	$p \wedge p = p$	$p \vee p = p$
Double Negation:	$\sim\sim p = p$	
DeMorgan's Rules:	$\sim(p \wedge q) = \sim p \vee \sim q$	$\sim(p \vee q) = \sim p \wedge \sim q$
Commutative Rules:	$p \wedge q = q \wedge p$	$p \vee q = q \vee p$
Absorption Rules:	$p \vee (p \wedge q) = p$	$p \wedge (p \vee q) = p$
Bound Rules:	$p \wedge 0 = 0 \qquad p \wedge 1 = p$	$p \vee 1 = 1 \qquad p \vee 0 = p$
Negation Rules:	$p \wedge (\sim p) = 0$	$p \vee (\sim p) = 1$

Problem 12 Scratch Paper

Do not detach the paper.

Problem 13 Scratch Paper

Do not detach the paper.