MAT 115: Exam II

Section: MWF 9:20-10:30 am

Date: 10/31/2018

Instructions:
You have **70** minutes for this exam. There are 10 + 1 problems. The total score is **100** plus extra 10 points from a bonus problem. Last problem is the set of set algebraic rules for your reference. Please work on those in which you have confidence. If you finish the exam early, please just leave your exam on the desk and I will collect it. I leave plenty of space on each page for your computation. One extra blank sheet is also attached at the end of this exam if you need more computation space. If your answer is incorrect but you show the computation process, then partial credits will be given. Please use the blank space in the exam and your time wisely.

First Name:

Last Name:

Score: **/100 +** **/10**

Problem 1 Simple Induction: 10pts (2+ 3 + 5)

Please use induction proof method to show that $\sum_{i=1}^{n} i^2 + 3i = \frac{n(n+1)(n+5)}{3}$

Base case:

Hypothesis:

Induction:

Problem 2 Euclidean Algorithm: 10pts (5 + 5)

Please find the GCD of the following using Euclidean algorithm (must show steps):
(a) 201810 and 199911

(b) 1071 and 462.

Problem 3 Euler function: 5+5 pts

Find the Euler number $\phi(N)$ for the following number

(a) $N = 3^2 * 5^2 * 4$

(b) $N = 28$

Problem 4 Lexical Order: 5 + 5 pts

Let $A = \{1, 2\}, B = \{u, v\}$, and $C = \{m, n\}$. Take the linear order on A to be numeric and the linear orders on B and C to be alphabetic. List the elements in each of the following sets in lexicographic order.
(a) $(A \times B) \times C$

(b) $A \times C \times B$

Problem 5 Set Proof: 10pts

Prove via algebraic rules or disapprove via a counterexample (such as Venn Diagram).
Note that $D - E = D \cap \sim E$
(a) If A and B are subsets of U and if $A \subseteq B$, then $A \cap (U - B) = \emptyset$.

(b) If A, B, and C are subsets of U, then $(A - B) - C = A - (C \cup B)$

Problem 6 Power Set: 5 + 5pts

Compare the following pairs of sets. If they are equal, please proof. If not, please show an counter exampl. Here the $\mathbb{P}(A)$ means the power set of A.
(a) $\mathbb{P}(A \cup B)$ and $\mathbb{P}(A) \cup \mathbb{P}(B)$

(b) $\mathbb{P}(A \times B)$ and $(\mathbb{P}(A) \times \mathbb{P}(B)$

Problem 7 Relation & Partition : 10 (6 + 3 + 1) pts

Let $A = \{1, 2, 3\}$ and $R = \{(1, 1), (2, 2), (3, 3), (2, 3), (3, 2), (1, 2), (2, 1)\}$.
(a) Is R an equivalence relation? (need to verify those three properties)

(b) Let $R_2 - R = \{(1, 3), (3, 1)\}$. We know R_2 is an equivalent relation. Please write out the equivalence classes (partitions) of A based on R_2

(c) $A = \{1, 2, 3, 5, 6\}$ and it can be partitioned by R_3 such that the partitions are $P_1 = \{1, 5\}, P_2 = \{2, 3, 5\}$ and $P_3 = \{6\}$. Please write out the relation R_3.

Problem 8 Equivalence Relation: 3 + 3 + 4 pts

Let the binary relation R act on the integer set $\mathbb{Z}$. Let d be two positive integers and R be defined that $(x, y) \in R$ if $d|(x^2 - y^2)$ where $x, y \in \mathbb{Z}$. Show R is an equivalence relation.

(a) Reflexive:

(b) Symmetric:

(c) Transitive:

Problem 9 Equivalence Relation: 10 pts

Define an equivalence relation R on the positive integers $A = \{2, 3, 4, ..., 21\}$ by mRn if the largest prime divisor of m is the same as the largest prime divisor of n. Please write out all the equivalence classes of R

Problem 10 Proof: Combination Formula : 5 + 5 pts

We talked about the combination. $C(n,k)$ means how many ways one can pick k distinct items out of n distinct items (grab k items at one time out of n items). It is know that $C(n,k) = \frac{n!}{(n-k)!k!}$ where $T! = 1 \times 2 \times 3 \times \cdots \times T$. Please show

(a) $\frac{n!}{(n-k)!k!} = (n \times (n-1) \times \cdots \times (k+1))/(n-k)!$

(b) Please show that $C(n-2,k-2)+C(n-2,k-1)+C(n-2,k-1)+C(n-2,k) = C(n,k)$ (provided you know $C(n-1,k-1)+C(n-1,k) = C(n,k)$ for all positive integers n,k and $k<n$).

Problem 11 Bonus: equivalence class : 10 pts

Let R be an equivalence relation acting on set $A = \{a_1, ..., a_m\}$ and let $S = \{S_1, ..., S_n\}$ be the set of equivalence classes based on R. Please show
(a) S is a set partition of A.

(b) Please explain why $S_i \cap S_j = \phi$ when $i \neq j$ and $1 \leq i, j \leq n$.

Problem 12 Set Algebraic Rules Sheet

Associative:	$(P \cap Q) \cap R = P \cap (Q \cap R)$	$(P \cup Q) \cup R = P \cup (Q \cup R)$
Distributive:	$P \cap (Q \cup R) = (P \cap Q) \cup (P \cap R)$	$P \cup (Q \cap R) = (P \cup Q) \cap (P \cup R)$
Idempotent:	$P \cap P = P$	$P \cup P = P$
Double Negation:	$\sim\sim P = P$	
DeMorgan:	$\sim(P \cap Q) = \sim P \cup \sim Q$	$\sim(P \cup Q) = \sim P \cap \sim Q$
Absorption:	$P \cup (P \cap Q) = P$	$P \cap (P \cup Q) = P$
Commutative:	$P \cap Q = Q \cap P$	$P \cup Q = Q \cup P$

Problem 13 Scratch Paper

Do not detach the paper.

Problem 14 Scratch Paper

Do not detach the paper.