MAT 115: Finite Math for Computer Science
Problem Set 1

Out: 09/14/2018 Due: 09/21/2018

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. Please turn in the hardcopy such that the TA can grade on it. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely. Late assignments will receive penalty.

First Name:

Last Name:

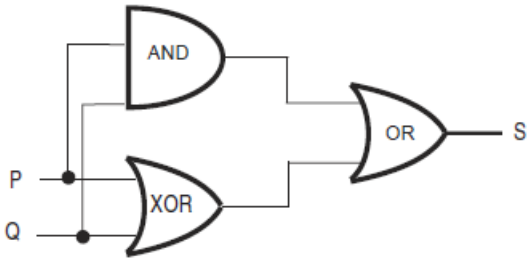
Group ID:

Score: **/100**

Bonus: **/10**

Problem 1 Circuits: 10pts

Please show the follwoing circuit is equivalent to XOR gate.



Problem 2 Proof: Algebraic Rules: 10pts

Is the function $(p \wedge (\sim (\sim p \vee q))) \vee (p \wedge q)$ equal to the function $p \vee q$?

Problem 3 Base Change: 20pts

Convert the following numbers
(a) BA0A (hex number into decimal form)

(b) 11012 (ternary number into decimal form)

(c) 2018 (base 9 number to decimal number)

(d) 125 (decimal number into ternary form (base 3))

Problem 4 Truth Table: 10pts

Make a truth table for $(\sim(\sim(p \wedge q) \wedge (\sim p \vee r)))\wedge \sim(q \vee \sim r \vee p)$

Problem 5 Proof: Algebraic Rules: 10pts

Is the function $(r \vee p) \wedge (\sim r \vee (p \wedge q)) \wedge (r \vee q)$ equla to the function $p \wedge q$

Problem 6 Boolean Functions: 15pts

Given a function $f : \{0,1\}^3 \to \{0,1\}$, please answer the following :
(a) Please show two input instances from the domain.

(b) What is the number of possible boolean functions f?

(c) Boolean functions can be found in many applied problems. Please briefly describe a problem (search on wikipedia) that is a direct application of boolean function and explain why this problem is important.

Problem 7 Representing Function: 10pts

Given a boolean function $f : \{0,1\}^3 \to \{0,1\}$, we can easily interpret it as $f(p,q,r) = s$ where $p,q,r \in \{0,1\}$ and $s \in \{0,1\}$. We have $f(0,0,0) = 1, f(0,1,0) = 1, f(1,0,0) = 0, f(1,1,0) = 0, f(0,0,1) = 1, f(0,1,1) = 1, f(1,0,1) = 0$ and $f(1,1,1) = 1$.
(1) Please derive the boolean function f.

(2) Please draw the circuit for this boolean function based on your answer in (1)

Problem 8 Circuits: 10+5pts

(a) For the full adder circuit on Boolean Function note, the operator for generating C is an OR gate. Normally, with intuition, it should be another half adder to add up c' and c''. What is the reason that we can use OR gate there to add up c' and c''?

(b) Furthermore, can we use XOR to replace that OR?

Problem 9 Bonus: 10pts

We learned about half adder and full adder in class. Let say if we are to add two k-bit binary numbers, p_1 and p_2.
(1) Draw your cicuit for solving this problem.

(2)Please explain what the gate complexity and the depth of the circuit are if you build the circuit based on full adder and let us assume each gate takes T time to run.

Problem 10 Practice Problems

For practice only. You do not have to turn in the solution.
Unit BF: 1.12, 1.14, 1.15, 2.1, 2.4, 2.5, 2.6