

MAT 115: Finite Math for Computer Science

Problem Set 2

Due: 10/01/2018

Instructions:

I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Group ID:

Score: /140

Problem 1 Binary and Ternary Operations: 10pts

Please compute the following

(a) $111001_2 \times 1010101_2$

(b) $12001001_3 \div 2020101_3$

Problem 2 Simple Proof :10pts

Given $n \in \mathbb{Z}$, show that $n^3 + 5$ is odd $\rightarrow n$ is even using

(a) a proof by contraposition

(b) direct proof

Problem 3 Predicate Logic: 10pts

$D = \{1, 3, 4, 5, 9, 121, 169, 196, 225, 289, 441\}$, $S(x) = (\sqrt{x} \in \mathbb{Z} \wedge \sqrt{x} + 2 \in \mathbb{P})$ where \mathbb{P} is the set of prime numbers. Let truth set $T = \{x \in D \mid S(x)\}$. Please show the elements inside the set T .

Problem 4 Proof: 10pts

Please show that for any integer m and n , $m^2 - n^2$ is even **if only if** $m - n$ is even.

(a) Given any integer m and n , if $m^2 - n^2$ is even, then $m - n$ is even

(b) Given any integer m and n , if $m - n$ is even, then $m^2 - n^2$ is even

Problem 5 Predicate: 12pts

Determine the truth value of each of these statements, given $x \in \mathbb{R}$. Furthermore, if the truth value is false, please show a counter example.

(a) $\exists x(x^2 = 2)$

(b) $\exists x(x^2 = -1)$

(c) $\forall x(x^2 + 1 \geq 2)$

(d) $\forall x(x^2 \neq x)$

Problem 6 Quantifier:10pts

Show that $\forall xP(x) \vee \forall xQ(x)$ and $\forall x(P(x) \vee Q(x))$ are not logically equivalent.

Problem 7 Nested Quantifier: 3*5pts

Let $Q(x)$ be the statement $x + y = x - y$, given $x, y, z \in \mathbb{Z}$. what are the truth value of the following expressions? And why?

(a) $\exists x \exists y Q(x, y)$

(b) $\exists y \forall x Q(x, y)$

(c) $\forall x \forall y Q(x, y)$

(d) $\forall x \forall y \exists z (z = (x + y)/2)$

(e) same as (d) but now $x, y, z \in \mathbb{R}$

Problem 8 Simple Induction: 2+3+5 pts

Please use induction proof method to show that $\sum_{i=1}^n i^2 + 3*i$ has a closed form as $\frac{n(n+1)(n+5)}{3}$

Base case:

Hypothesis:

Induction:

Problem 9 Mersenne Number: 5 + 3 + 5 + 5 pts

We talked about Mersenne Number and Mersenne Prime Number in class. Please reserach on the internet and answer the following:

(a) List 2 applications of prime numbers in computer science

(b) What is the current biggest prime number? Is it a Mersenne Prime?

(c) Based the in class exercise that a Mersenne prime is $M_p = 2^n - 1$ where n is a prime. Based on the answer you have in (b), what is the current largest even perfect number

(d) Why n cannot be an even number for M_p (assuming $n > 2$)?

Problem 10 Propositional Logic: 10pts

Is the statement form $((\sim p \wedge q) \wedge (q \vee r)) \wedge \sim q \wedge r$ a tautology, contradiction or neither?
Please use algebraic rules.

Problem 11 Propositional Logic: 10pts

Is $(p \wedge \sim q) \wedge (\sim p \vee q) \wedge r$ a tautology, contradiction or neither? Please use algebraic rules.

Problem 12 Floor Ceiling Functions: 2+3 pts

Please compute the following:

(a) $\lceil (\lfloor 3.85 \rfloor * \lceil -6.7 \rceil + 2.5) \rceil * 3$

(b) $\lfloor -3.5 \rfloor * \lceil 5.6 \rceil + \lceil (2.5 * 3.5) \rceil$

Problem 13 Modulo Functions: 10pts

Prove that for all integers $n > 1$, $n^2 - 3 \not\equiv 0 \pmod{4}$. [Hint: Do by cases]

Problem 14 Practice Problems

For practice only. You do not have to turn in the solution.

Unit Lo: 1.10, 1.19, 2.13, 2.19.11, 1.1

Unit NT: 1.6(a)(b), 1.14, 1.19, 1.25(a), 1.28