MAT 115: Finite Math for Computer Science
Problem Set 3

Due: 10/22/2018

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Group ID:

Score: **/115 + /10 (bonus)**

Problem 1 Simple Induction: 2+3+5 pts

Please use induction proof method to show that $\sum_{i=1}^{n} i$ has a closed form as $\frac{n(n+1)}{2}$

Base case:

Hypothesis:

Induction:

Problem 2 Euclidean algorithm: 5+5 pts

Us the Euclidean algorithm to find all common divisors of

(a) 1001 and 544

(b) 252 and 180

Problem 3 Euler function: 5+5+5 pts

Us the Euler number for the following number

(a) $N = 2^4 * 5 * 7$

(b) $N = 4 * 10 * 15$

(c) $N = 21$

Problem 4 Use of Subtraction Trick: 20 pts

Prove each of the following identities from the basic algebraic rules for sets. You may want to use the fact that $D - E = D \cap \sim E$ [Here the $\sim$ means the complement].
(a) If A and B are subsets of U, then $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

(b) If A, B, and C are subsets of U, then $(A - B) - C = (A - C) - B$

Problem 5 Disprove via Venn Diagram: 5+5 pts

Show the following statement is wrong via Ven Diagram example.
(a) For all sets A, B and C, we hav $(A \cup B) \cap C = A \cup (B \cap C)$

(b) If A, B, and C are subsets of U, then $(A - B) - C = A - (B - C)$

Problem 6 Ordered Set: 5+5 pts

Let $A = w, x, y$ and $B = a, b, c$. List the elements in each of the following sets in lexigraphic order:
(a) $B \times A$

(b) $B \times (A \times B)$

Problem 7 Combinations: 5+5 pts

Compute
(a) $C(14, 5)$

(b) $C(23, 6)$

Problem 8 Power Set: 5 + 5 + 5 + 5 pts

$A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$, compute the following:
(a) Power set of $A - B$.

(b) Power set of $B - A$

(c) Let $C = A \cup B$. What is $|C|$?

(d) $P(P(P(\{1, 2\}))$ here P means power set.

Problem 9 Partition: 10 pts

How many refinements are there of the partition $\{\{1,3,5\},\{2,6\},\{4,7,8,9\}\}$? Please explain.

Problem 10 Bonus: 10 pts

Let $S = \{1, 2, \cdots, n\}$. Let S_1 be the set of all subsets of that that contain 1. Let T_1 be the set of all subsets of S that do not containt 1. Show that $T_1 = S_1 = 2^{n-1}$.

Problem 11 Practice Problems

For practice only. You do not have to turn in the solution.
Unit SF: 1.6, 1.8, 1.9, 1.14, 2.19