MAT 115: Finite Math for Computer Science
Problem Set 4

Due: 11/02/2018

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Group ID:

Score: **/130 + /10 (bonus)**

Problem 1 Power Set: 15pts

Compare the following pairs of sets. If they are equal, please proof. If not, please show an counter exampl. Here the $\mathbb{P}(A)$ means the power set of A.
(a) $\mathbb{P}(A \cup B)$ and $\mathbb{P}(A) \cup \mathbb{P}(B)$

(b)$\mathbb{P}(A \cap B)$ and $\mathbb{P}(A) \cap \mathbb{P}(B)$

(c) $\mathbb{P}(A \times B)$ and $(\mathbb{P}(A) \times \mathbb{P}(B)$

Problem 2 Relation & Partition : 10 + 5 + 5 pts

Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 4), (4, 1), (2, 3), (3, 2)\}$.
(a) Is R an equivalence relation? (need to verify those three properties)

(b) What are the equivalent classes (partitions) of A based on R

(c) $A = \{1, 2, 3, 4, 5, 6\}$ and it can be partitioned by R_2 such that the partitions are $P_1 = \{1, 5\}, P_2 = \{2, 4\}$ and $P_3 = \{3, 6\}$. Please write out the relation R_2.

Problem 3 Equivalence Relation: 20 (5+ 10 + 5) pts

Define an equivalence relation R on the positive integers $A = \{5, 6, 7, ..., 15\}$ by mRn, i.e. $m, n \in A$. $(m, n) \in R$ when the largest prime divisor of m is the same as the largest prime divisor of n.

(a) Write out all the pairs in R

(b) Is R an equivalence relation? Why (explain reflexivity, symmetry and transitivity) ?

(c) Based on (a), what is the number of equivalence classes of R?

Problem 4 Proof: Combinatorial 15: 5 + 10 pts

We talked about the combination. $C(n, k)$ means how many ways one can pick k distinct items out of n distinct items (grab k items at one time out of n items). It is know that $C(n, k) = \frac{n!}{(n-k)!k!}$ where $T! = 1 \times 2 \times 3 \times \cdots \times T$ for any positive integer T. Please show
(a) $\frac{n!}{(n-k)!k!} = (n \times (n-1) \times \cdots \times (n-k+1))/k!$

(b) Please show that $C(n-1, k-1) + C(n-1, k) = C(n, k)$ [For instance, you can verify that $C(4, 2) + C(4, 3) = C(5, 3)$].

Problem 5 Subsets Defintions: 5*3pts

Answer the following about the $\in$ and $\subseteq$ operators.
(a) Is $\{1,2\} \subset \{\{1,2\},\{3,4\}\}$?

(b) Is $\{2\} \in \{1,2,3,4\}$?

(c) Is $\{3\} \in \{1,\{2\},\{3\}\}$

(d) Is $\{1,2\} \subseteq \{1,2,\{1,2\},\{3,4\}\}$? and $\{1,2\} \in \{1,2,\{1,2\},\{3,4\}\}$?

(e) Is $1 \in \{\{1\},\{2\},\{3\}\}$?

Problem 6 Subset Proof : 10pts

A, B and C are subsets of U, show that if $A \subseteq C$ and $B \subseteq C$ implies $(A \cup B) \subseteq C$. [Hint: Use definition of subset and then the union of two sets has three parts: 1. in A only 2. in B only, 3. in both A and B.

Problem 7 Equivalence Relation: 15pts

Let the binary relation R act on the integer set $\mathbb{Z}$. Let d, k be two positive integers and R be defined that $(x, y) \in R$ if $d|(x^k - y^k)$ where $x, y \in \mathbb{Z}$. Show R is an equivalence relation.
(a) Reflexive:

(b) Symmetric:

(c) Transitive:

Problem 8 Types of Functions: 20 (5 + 3 + 5 + 3 + 4)pts

Let A and B be finite sets and $f : A \to B$. Prove the following claims. Some are practically restatements of the definitions, some require a few steps.
(a) If f is an injection, then $|A| \leq |B|$.

(a-1) Show an example that $|A| \leq |B|$, but f is not an injection.

(b) If f is a surjection, then $|A| \geq |B|$.

(b-1) Show an example that $|A| \geq |B|$, but f is not an surjeciton.

(c) If f is a bijection, then $|A| = |B|$.

Problem 9 Bonus: equivalence class : 10 pts

Let R be an equivalence relation acting on set $A = \{a_1, ..., a_m\}$ and let $S = \{S_1, ..., S_n\}$ be the set of equivalence classes based on R. Please show
(a) S is a set partition of A.

(b) Please explain why $S_i \cap S_j = \varphi$ when $i \neq j$ and $1 \leq i, j \leq n$.