MAT 115: Finite Math for Computer Science
Problem Set 5

Due: 12/07/2018

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Group ID:

Score: **/115 + /20 (bonus)**

Problem 1 Permutation: the length of the cycle 3+4+4+4 pts

All the permutations given below are in cycle form. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$
(a) Please compute $((1, 3), (2, 5, 4), (6), (7))^{61}$

(b) $f : A \to A$ is a permuation and $f = (3, 5, 4, 2, 1, 7, 6), g = (1, 4, 6, 3, 2, 7, 5))$.
(1) f in cycle form

(2) $(f \circ g)^{-1}$ in 2 line form

(3) What is the period of $(f \circ g)^{-1}$

Problem 2 Pigeonhole Concept (10pts)

As seen in class, let $A = \{a_1, a_2, \cdots, a_t\}$ be a set containing t distinct positive integers. Suppose we expect to have $a_i + a_j + a_k = a_l + a_m + a_n$ occur under the modulo function N where (1) $1 \leq i, j, k, l, m, n \leq t$ and i, j, k are distinct numbers (2) l, m, n are distinct integers and (3) $(i, j, k) \neq (l, m, n)$. Please find the smallest postive number t when $N = 91$ [Hint: Translate via $C(t, 3)$].

Problem 3 Permutation Application: 5 + 5 pts

(a) How many ways are there for eleven women and five men to stand in a line so that no two men stand next to each other?

(b) Same as (a) but now the women are labeled $\{A, B, C, D, E, F, G, H, I, J, K\}$ and the men are labeled $\{\alpha, \beta, \gamma, \delta, \eta\}$

Problem 4 Permutation: With and Without Repetition (3+3+9=15 pts)

We are interested in forming 3 letter words using the letters in onondagaagadnono. For the purpose of the problem, a word is any **list** of letters. Please answer the following:
(a) How many words can be made with no repeated letters?

(b) How many words can be made with unlimited repetition allowed?

(c) How many words can be made if repeats are allowed but no letter can be used more than it appears in onondagaagadnono?

Problem 5 Permutation: No Repetition 2*7 = 14

We work with the ordinary alphabet of **28-letters** (A-Z plus τ, λ. Please solve the following:
(a) Define a 5-letter word to be any list of 5 letters that contains *at least* one of the vowels A, E, I, O and U. How many 5-letter words are there?

(b) We can solve (a) in one single step but we can do it in 6 steps. Here are the steps:
(b-1) How many 5-letter words with exactly 1 vowel

(b-2) How many 5-letter words with exactly 2 vowels

(b-3) How many 5-letter words with exactly 3 vowels

(b-4) How many 5-letter words with exactly 4 vowels

(b-5) How many 5-letter words with exactly 5 vowels

(b-6) Your sum from b-1 till b-5 is? The result should be equivalent to (a)

Problem 6 Combinatorial : 6 pts

A professor writes 40 discrete mathematics true/false questions. Of the statements in these questions, 17 are true. If the questions can be positioned in any order, how many different answer keys are possible?

Problem 7 Pigeon Holes: 5 pts

Assuming that no one has more than 1,000,000 hairs on the head of any person and that the population of New York City was 8,008,278 in 2010, show there had to be at least nine people in NewYork City in 2010 with the same number of hairs on their heads.

Problem 8 Stirling Number: 10 (required) +10(bonus) + 10 (bonus) pts

For $n > k > 0$, the Stirling number of the 2nd kind is $S(n, k) = S(n-1, k-1) + k \times S(n-1, k)$.
(a) Please find the close form for $S(n, 3)$ provided you know that $S(n, 2) = 2^{n-1} - 1$, $S(n, k) = S(n-1, k-1) + k \times S(n-1, k)$. (Hint: recursive calls add up the exponents)

(b) Please prove (a) using Induction (bonus)

(c) In (a), assume that you know $S(n, 2) = 2^{n-1} - 1$ and it is obvious that $S(n, 1) = 1$. Please prove that $S(n, 2) = 2^{n-1} - 1$.

Problem 9 Probability + Counting ($3\times$ 3 $\times$ 2 = 18 pts)

An urn A contains eleven labeled balls, labels 1,2, $\cdots$, 11. An urn B contains six labeled balls, labels 1,2, $\cdots$,6.
(a) Two balls are drawn, one from A and one from B. What is the sample space? What is the probability that the sum of the labels on the balls is odd? What is the probability that the sum of the labels on the balls is 9?

(b) Two balls are drawn one after the other without replacement and the order matters from urn A. What is the sample space? What is the probability that the sum of the labels on the balls is odd and the first ball number must be greater than the 2nd ball number)? What is the probability that the sum of the labels on the balls is 9?

(c) Two balls are drawn from urn B one after the other with replacement and the order matters. What is the sample space? What is the probability that the sum of the labels on the balls is even? What is the probability that the sum of the labels on the balls is 10?

Problem 10 Graph: 4 + 4 + 4 pts

For a graph $G = (V, E)$, let $d(v)$ be the degree of the vertices $v \in V$.
(a) A complete bipartite **undirected** graph $G = (V_1, V_2, E)$ is a graph where each node v_i in V_1 has one edge to each node v_j in V_2. Each v_j in V_2 has one edge to each node v_i in V_1. Given $|V_1| = n_1, |V_2| = n_2$, that means $d(v_i) = n_2, \forall v_i \in V_1$ and $d(v_j) = n_1, \forall v_j \in V_2$. What is $|E|$?

(b) Please draw the the following **directed** graph $G_1 = (V, E)$ where $V = \{a, b, c, d, e\}$ and $E = \{(a, d), (a, e), (b, d), (b, e), (c, d), (c, e), (d, a), (d, b), (d, c), (e, a), (e, b), (e, c)\}$.

(c) Is it possible for the above graph G_1 to have a cycle and why?