CS 528: Quantum Computation
Assessment Exam

MW: 12:00 - 1:15 pm

10/06/2019

Instructions:
I leave plenty of space on each page for your computation. You have 75 mninutes for this test and the total score is 75 + 10 (bonus). Please use your time wisely.

First Names:

Group ID:

Score: **/75+ 10**

Problem 1 Formula : 10 pts [Bonus]

Please show that $H^{\otimes n}|j\rangle = \frac{1}{\sqrt{2^n}} \sum_{i=0}^{2^n-1} (-1)^{ij} |i\rangle$ **via induction**. Ther term ij is the inner product of vectors $|i\rangle$ and $|j\rangle$ (they are both $\in \mathbb{R}^n$).

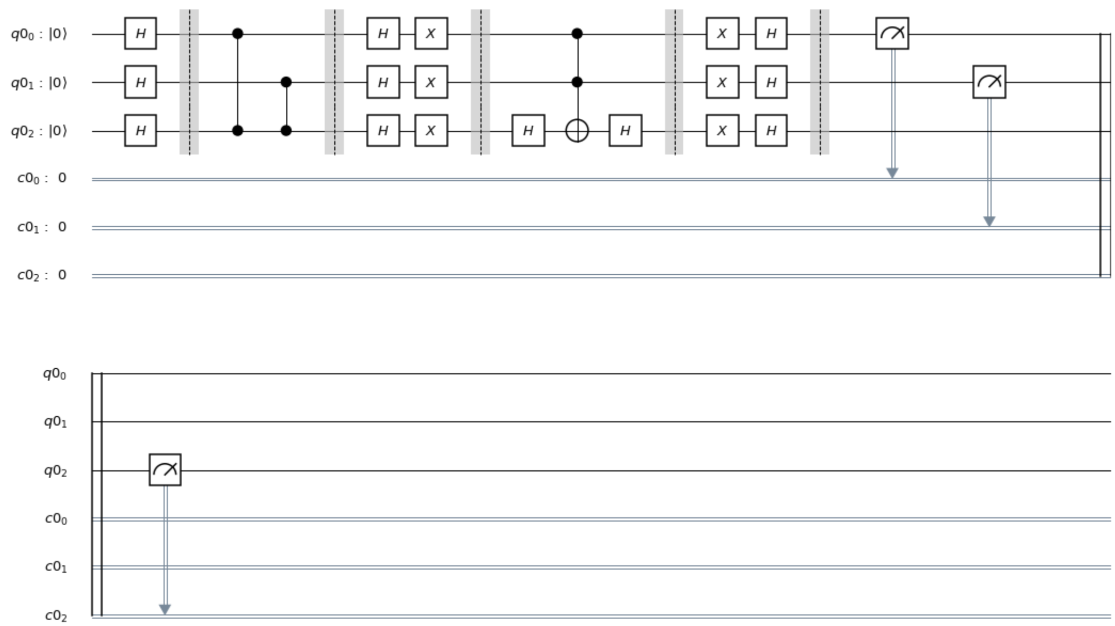
Problem 2 Qiskit + Deutsch Jozsa : 10 + 5 pts

(a) Please write out the Qistkit source code implementing the Deutsch-Joza

(b) Please show the circuit implementing the Deutsch-Joza

Problem 3 Grover : 10 + 10 pts

(1) Please mark the area of circuit that is performing the 2nd reflection $2|\psi\rangle\langle\psi| - I$[2pts]. And what is missing [**need to justify**] and why this missing part will not affect the Grover search result[8pts]?



(2) Please show that for a generic Grover, assuming the initial state

$$|\psi\rangle = \frac{1}{\sqrt{N}} \sum_i |i\rangle = \sin\theta|G\rangle + \cos\theta|B\rangle$$

Please derive and verify the G operator in the the $|G\rangle, |B\rangle$ basis.

Problem 4 Simon's algorithm: 5+5+5 pts

Suppose we run Simon's algorithm on the following input x (with $N = 8$, and hence $n = 3$):

$$x_{000} = x_{101} = 000 \qquad x_{001} = x_{100} = 001 \tag{1}$$

$$x_{010} = x_{111} = 010 \qquad x_{011} = x_{110} = 011 \tag{2}$$

Note that s is a 2-to-1 and $x_i = x_{i \oplus 111}$ for all $i \in \{0,1\}^3$, so $s = 101$

(a) Give the state after measuring the second register (suppose $|001\rangle$)

(b) Give the state after final Hadamards

(c) Suppose the first run of the algorithm gives $j = 001$, and the second gives $j = 101$. Show that assuming $s \neq 000$, those two runs of the algroithm already determine s.

Problem 5 Analysis Technique Proof: 5 pts

Show that $e^{iAx} = cos(x)I + isin(x)A$ where $A^2 = I$ and x is some real number.

Problem 6 QFT : 5+5 pts

(1) Please show that

$$QFT_N|x\rangle = \frac{1}{\sqrt{N}}(|0\rangle + e^{\frac{2\pi i x}{2}}|1\rangle) \otimes \cdots \otimes (|0\rangle + e^{\frac{2\pi i x}{2^n}}|1\rangle)$$

where $|x\rangle = |x_1 \cdots x_n\rangle$ and $N = 2^n$

(2) Please draw the circuit where $n = 3$.

Problem 7 Superdense Coding: 5 + 5 pts

(1) Draw the circuit for superdense coding

(2) Verify how Alice sends the message 10 to Bob using the above circuit.