# CS 528: Quantum Computation
# Problem Set 1

MW: 12:00 - 1:15 pm

Out: 09/16/2019 Due: 09/30/2019

**Instructions:**
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. Please directly hit the point when solving a problem. Cumbersome description might receive fewer credits, even it is correct. If your answer is incorrect but you your logic is on the right track, then partial credits will be given. Please staple your solution and use the space wisely.

**First Names**:

**Group ID**:

**Score**: **/130 + 10**

## Problem 1 Schroedinger Equation: 10 pts

When we have $|\psi(t)\rangle = U|\psi(0)\rangle$, where $U = e^{\frac{-iHt}{h}}$, then $|\psi(t)\rangle$ is a solution to the Schroedinger equation

## Problem 2 Formula : 20 pts

Please show that $H^{\otimes n}|j\rangle = \frac{1}{\sqrt{2^n}} \sum_{i=0}^{2^n-1} (-1)^{ij} |i\rangle$. Ther term $ij$ is the inner product of vectors $|i\rangle$ and $|j\rangle$ (they are both $\in \mathbb{R}^n$).

## Problem 3 Imperfect Deutsch: 20pts

In the Deutsch and Deutsch-Jozsa algorithms, when we consider $U_f$ as a single-qubit operator $\hat{U}_{f(x)}$ , $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$ is an eigenstate of $\hat{U}_{f(x)}$, whose associated eigenvalue gives us the answer to the Deutsch problem. Suppose we did not prepare $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$ but $|\mathbf{0}\rangle$ instead in the target qubit and we just run the same circuit on that configuration.

(a) Please compute and explain the probability that we get the right answer for the modified Deutsch algorithm circuit when the function is constant.

(b) Please compute and explain the probability that we get the right answer for the modified Deutsch algorithm circuit when the function is balanced.

## Problem 4 Imperfect Deutsch Josza Algorithms: 20+10pts

Same as previous problem but for Deutsch-Jozsa algorithms.
(a) Please compute and explain the probability that we get the right answer for the modified Deutsch-Jozsa algorithm circuit when the function is constant.

(b) Please compute and explain the probability that we get the right answer for the modified Deutsch-Jozsa algorithm circuit when the function is balanced.

(c) Bonus [5+5]: If you can also correctly identify the value of the ancillary bit in the (a) and (b).

## Problem 5 Experiment with IBM Qiskit: 60pts

In this section, we run the IBM Qiskit simulation. You can refere to https://community.qiskit.org/textbook/ch-algorithms/deutsch-josza.html for the code for Deutsch-Jozsa algorithm implmentation. Let say we are to implement the imperfect Deutsch Jozsa algorithm with the following setting: (seee next page)

(I) $n = 4$ and a balanced function F that maps the input to 1 when there is even number of 1's in the input string, otherwise it maps the input to 0.
(a) Please show the code implementing this modified Deutsch-Joza

(b) Please show the circuit implementing this modified Deutsch-Joza

(c) Simulate the experiment 1024 times. Does the result echo to your theoretical computatin in problem 4 ?

(II) $n = 4$ and a constant function F that maps all the input to 1
(d) Please show the code implementing this modified Deutsch-Joza

(e) Please show the circuit implementing this modified Deutsch-Joza

(f) Simulate the experiment 1024 times. Does the result echo to your theoretical computatin in problem 4 ?