CS 528: Quantum Computation
Problem Set 2

MW: 12:00 - 1:15 pm

Out: 10/21/2019 Due: 11/06/2019

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. Regarding implementation, please make sure the code and the circuit you implemented via IBM Q are correctly attached right behind each problem. Please directly hit the point when solving a problem. Cumbersome description might receive fewer credits, even it is correct. If your answer is incorrect but you your logic is on the right track, then partial credits will be given. Please staple your solution and use the space wisely.

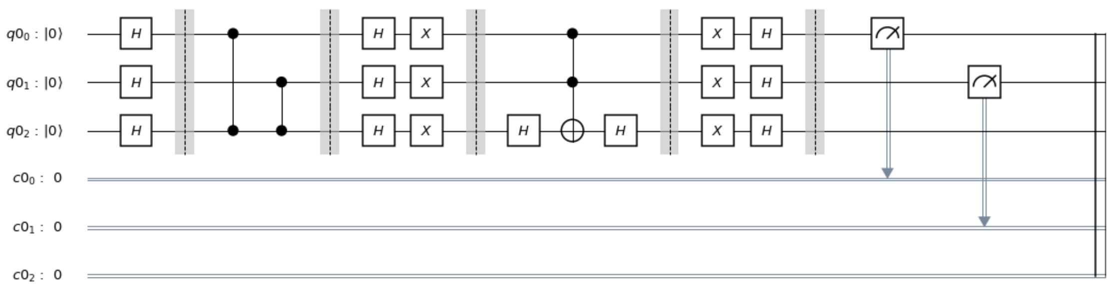
First Names:

Group ID:

Score: **/100**

Problem 1 Grover : 10 + 20 +10 pts

(1) On the IBM Qiskit site, it has the implementation https://community.qiskit.org/textbook/ch-algorithms/grover.html of Grover (see below). Which part of the circuit is performing the 2nd reflection that $2|\psi\rangle\langle\psi| - I$? And what is missing and why this missing part will not affect the Grover search result?



(2) Implement the Grover while this time we want the solution state to be only $|111\rangle$

(3) Please show that for a generic Grover, assuming the initial state

$$|\psi\rangle = \frac{1}{\sqrt{N}} \sum_i |i\rangle = \sin\theta|G\rangle + \cos\theta|B\rangle$$

Please derive and verify the G operator in the the $|G\rangle, |B\rangle$ basis.

Problem 2 Simon's algorithm: 30 pts

Suppose we run Simon's algorithm on the following input x (with $N = 8$, and hence $n = 3$):

$$x_{000} = x_{111} = 000 \qquad x_{001} = x_{110} = 001 \tag{1}$$
$$x_{010} = x_{101} = 010 \qquad x_{011} = x_{100} = 011 \tag{2}$$

Note that s is a 2-to-1 and $x_i = x_{i \oplus 111}$ for all $i \in \{0,1\}^3$, so $s = 111$
(a) Give the initial state of Simon's algorithm

(b) Give the state after the first Hadamard transforms on th efirst 3 qubits

(c) Give the state after measuring the second register (suppose $|001\rangle$)

(d) Give the state after final Hadamards

(e) Why does a measurement of the first 3 qubits of the final state give information about s?

(f) Suppose the first run of the algorithm gives $j = 001$, and the second gives $j = 101$. Show that assuming $s \neq 000$, those two runs of the algroithm already determine s.

Problem 3 QFT + QPE : 30 pts

(1) Please show that $QFT_N|x\rangle = \frac{1}{\sqrt{N}}(|0\rangle + e^{\frac{2\pi ix}{2}}|1\rangle) \otimes \cdots \otimes (|0\rangle + e^{\frac{2\pi ix}{2^n}}|1\rangle)$ where $|x\rangle = |x_1 \cdots x_n\rangle$ and $N = 2^n$

(2) Please draw the circuit using IBM Q where $n = 3$.

(2) Given the eigenstate $|\mu\rangle$ for the unitary U where $U|\mu\rangle = e^{2\pi i\theta}$ where $\theta = 0.\theta_1, ...\theta_n$ in binary representation. Please draw the circuit for QPE using the circuit you designed in previous subproblem. We expect the first qubit gives us θ_1, not θ_n.