# MAT 115: Exam 1

**Instructions:**

You have 70 minutes for this exam. There are 8+1 problems. Calculators are allowed. The total score is 100+10 points with perfect score of 100 and a bonus of 10 points. Please work on those in which you have confidence. If you finish the exam early, please just leave your exam on the desk and I will collect it. I leave plenty of space on each page for your computation. One extra blank sheet is also attached at the end of this exam if you need more computation space. If your answer is incorrect but you show the computation process, then partial credits will be given. Please use your time and space wisely.

**First Name**:

**Last Name**:

**Score**: **/100**

**Bonus: /10**

**Total: /110**

# Problem 1 Truth Table + Circuit design (10 + 5pts)

(a) Make a truth table for $f = [\{(\sim p) \wedge \sim r\} \vee \{(\sim q) \oplus q\}] \wedge [(\sim p) \vee \{q \vee r\}]$

| p | q | r | f |
|---|---|---|---|
| 0 | 0 | 0 | |
| 0 | 0 | 1 | |
| 0 | 1 | 0 | |
| 0 | 1 | 1 | |
| 1 | 0 | 0 | |
| 1 | 0 | 1 | |
| 1 | 1 | 0 | |
| 1 | 1 | 1 | |

(b) Construct a combinatorial circuit using NOT gates, OR gates and AND gates that produces the output as f does from input bits $p, q,$ and $r$. Please make sure your circuit is **as short as possible**.

## Problem 2 Base Change: 15 (5+5+5) pts

(a) Please convert $2016_{10}$ into a base 16 number. You must show the computation

(b) Please convert $AABA_{16}$ into a base 8 number.

(c) Please show the result of $1252_{10} + ABAA_{16}$ in base 13 number.

## Problem 3 : Simple Proof :10pts

Given $n \in \mathbb{Z}$, show that $n^3 + 5$ is odd $\rightarrow n$ is even using
(a) a proof by contraposition

(b) a proof by contradiction

## Problem 4 Logic: Quantifiers (5+5+5 pts)

Is the following statement true or false? Explain why. If you disprove, you must provide the counter example.
(a) $\exists x \in D, (P(x) \wedge Q(x))$ is the same as $(\exists x \in D, P(X)) \wedge (\exists x \in D, Q(x))$

Let Q(x) be the statement, given x, y, z $\in$ Z and x = 0. what is the truth value of the following expressions? If true, find the corresponding values.

(a) $\exists x \; \exists y \; Q\,(x, y){:} = (x + y)/x = (x - y),$

(b) $\exists Z \; \forall x \; \forall y \; Q\,(x, y) := ((x + y)/x)z = (x - y)z,$

## Problem 5: Proof: 10pts

Please show that for any integer $m$ and $n$, $m^2 - n^2$ is even **if only if** $m - n$ is even.

(a) Given any integer $m$ and $n$, if $m - n$ *is* even, then $m^2 - n^2$ is even

(b) Given any integer $m$ and $n$, if $m^2 - n^2$ is even, then $m - n$ *is* even

# Problem 6 Floor, Modulo, Ceiling functions: 5 * 2 pts

Please compute the following:

(a) $\lceil(\lfloor 3.85 \rfloor * \lceil -6.7 \rceil + 2.5)\rceil * 3$

(b)X $(\lfloor -3.5 \rfloor * \lceil 5.6 \rceil) + \lceil(2.5 * 3.5)\rceil$

## Problem 7: Propositional Logic: 10pts

Is $(p \wedge \sim q) \wedge (\sim p \vee q) \wedge r$ a tautology, contradiction or neither? Please use algebraic rules.

## Problem 8: Conditional Statements: 3*(3+2): 15 pts

With this propositional logic that if x is a pentagon, then x is a polygon, we can simply let P = x is a pentagon, Q = x is a polygon, then we know that P is not equal to Q. With this statement, please show its conditional statements

(a) Contrapositve Statement. Is the statement true or not and why?

(b) Converse Statement. Is the statement true or not and why?

(c) Inverse statement. Is it a true statement and why?

## Problem 9: Bonus: Mersenne Prime: 2+8 pts

(a) What is a Mersenne Prime and what is a perfect number?

(b)If $2^k$-1 is a prime number, then show $2^{k-1}(2^k - 1)$ is an even perfect number

## BASIC ALGEBRAIC RULES:

**Theorem 2 (Algebraic rules for Boolean functions)** *Each rule states that two different-looking Boolean functions are equal. That is, they look different but have the same table.*

| | | |
|---|---|---|
| *Associative Rules:* | $(p \wedge q) \wedge r = p \wedge (q \wedge r)$ | $(p \vee q) \vee r = p \vee (q \vee r)$ |
| *Distributive Rules:* | $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$ | $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$ |
| *Idempotent Rules:* | $p \wedge p = p$ | $p \vee p = p$ |
| *Double Negation:* | $\sim\sim p = p$ | |
| *DeMorgan's Rules:* | $\sim(p \wedge q) = \sim p \vee \sim q$ | $\sim(p \vee q) = \sim p \wedge \sim q$ |
| *Commutative Rules:* | $p \wedge q = q \wedge p$ | $p \vee q = q \vee p$ |
| *Absorption Rules:* | $p \vee (p \wedge q) = p$ | $p \wedge (p \vee q) = p$ |
| *Bound Rules:* | $p \wedge 0 = 0 \qquad p \wedge 1 = p$ | $p \vee 1 = 1 \qquad p \vee 0 = p$ |
| *Negation Rules:* | $p \wedge (\sim p) = 0$ | $p \vee (\sim p) = 1$ |

**SCRATCH WORK**