MAT 115: Exam 2

Instructions:

You have 70 minutes for this exam. There are 7+1 problems. Calculators are allowed. The total score is 100+10 points with perfect score of 100 and a bonus of 10 points. Please work on those in which you have confidence. If you finish the exam early, please just leave your exam on the desk and I will collect it. I leave plenty of space on each page for your computation. One extra blank sheet is also attached at the end of this exam if you need more computation space. If your answer is incorrect but you show the computation process, then partial credits will be given. Please use your time and space wisely.

First Name:

Last Name:

Score: **/100**

Bonus: **/10**

Total: **/110**

Problem 1 GCD (10 pts)

Use the Euclidean Algorithm to find the GCD of the following:

(a) 1001 and 544

(b) 3510 and 652

Problem 2 GCD (10 Points)

Find all common divisors of 252 and 180 using the Euclidean algorithms

Problem 3 : Euler Function: (10 +5 pts)

(a) Given n = 162, find φ(n)

(b) Given n = 210, find φ(n)

Problem 4 GCD and Linear Combination (15 Points)

Prove each of the following identities from the basic algebraic rules for sets. You may want to use the fact that $D - E = D \cap E^c$.

If A and B are subsets of U, then $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

Problem 5 Sets (20 points)

Let A = {w, x, y, z} and B = {a, b}. Take the linear orders on A and B to be alphabetic order. List the elements in each of the following sets in lexicographic order.

(a) $A \times B$ and $B \times A$

(b) Each of the following statements about subsets of a set U is FALSE. Draw a Venn diagram to represent the situation being described. In each case case, show that the assertion is false by specializing the sets.

For all sets A, B, and C, $(A \cup B) \cap C = A \cup (B \cap C)$.

Problem 6 Functions (15 Points)

Let A be a set with m elements and B be a set with n elements. SF Sets and Functions

(a) How many relations are there on $A \times B$?

(b) How many functions are there from A to B?

Problem 7: Subset: (3 * 5 = 15pts)

Answer the following about ∈ and ⊆ operators.

(a) Is {1, 2} ∈ {{1, 2}, {3, 4}}?

(b) Is {2} ∈ {1, 2, 3, 4}?

(c) Is {3} ∈ {{1, 2}, {3}, {4}}?

Problem 8: Bonus (10 pts)

Show by induction: $3 \mid n^3 - n$ where n is a positive integer

BASIC ALGEBRAIC RULES:

Theorem 2 (Algebraic rules for Boolean functions) *Each rule states that two different-looking Boolean functions are equal. That is, they look different but have the same table.*

Associative Rules:	$(p \wedge q) \wedge r = p \wedge (q \wedge r)$	$(p \vee q) \vee r = p \vee (q \vee r)$
Distributive Rules:	$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$
Idempotent Rules:	$p \wedge p = p$	$p \vee p = p$
Double Negation:	$\sim\sim p = p$	
DeMorgan's Rules:	$\sim(p \wedge q) = \sim p \vee \sim q$	$\sim(p \vee q) = \sim p \wedge \sim q$
Commutative Rules:	$p \wedge q = q \wedge p$	$p \vee q = q \vee p$
Absorption Rules:	$p \vee (p \wedge q) = p$	$p \wedge (p \vee q) = p$
Bound Rules:	$p \wedge 0 = 0 \qquad p \wedge 1 = p$	$p \vee 1 = 1 \qquad p \vee 0 = p$
Negation Rules:	$p \wedge (\sim p) = 0$	$p \vee (\sim p) = 1$

SCRATCH WORK