MAT 115: Finite Math for Computer Science

Problem Set 2

Due: 4th Oct,2019.

- **Instructions:** I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

- **Group Details**: A group can have a maximum of 3 people. Kindly provide your batch or group details and any kind of changes are not accepted once fixed.

- **Late Assignment Rules:**
  1-2 days delay: - 20 % of total score

  3-5 days delay: - 30 % of total score

  >5 days delay: - 50 % of total score

  > 2 weeks delay: 0 points

First Name:

Last Name:

Group ID:

Score: /100

Problem 1: Propositional Logic. (15 Points)

Let h = "he is happily married," and w = "he is wealthy," and s = "he is smart." Write the following statements in symbolic form:

(a) He is happily married and wealthy but not smart.

(b) He is not wealthy, but he is happily married and smart.

(c) He is neither happily married, nor wealthy, nor smart.

Problem 2: Propositional Logic (10 Points)

Construct a truth table for $\sim p \vee (p \wedge \sim q) \Rightarrow q$ where p and q are the Boolean variables

Problem 3: Logic (10 Points)

Is the statement form $(p \wedge \sim q) \wedge (\sim p \vee q) \wedge r$ a tautology, contradiction, or neither?

Problem 4: Predicate Logic (15 points)

Consider the statement "A large income is a necessary condition for happiness."

(a) Let P be the set of people. For x ∈ P, let L(x) indicate that x has a large income and H(x) that x is happy. Rewrite the given statement using the notation of logic rather than the English language.

(b) Write the statement in ordinary English, without using "necessary" or "sufficient."

(c) Write the negation of the statement in logic notation. Move the negation inside the statement as far as possible.

(d) Write this negation in ordinary English, without using "necessary" or "sufficient."

Problem 5: Choose the correct answer (10 Points)

The statement form $(p \Leftrightarrow r) \Rightarrow (q \Leftrightarrow r)$ is equivalent to

(Give proper explanation or valid proof in determining the answer)

(a) $[(\sim p \vee r) \wedge (p \vee \sim r)] \vee \sim [(\sim q \vee r) \wedge (q \vee \sim r)]$

(b) $\sim [(\sim p \vee r) \wedge (p \vee \sim r)] \wedge [(\sim q \vee r) \wedge (q \vee \sim r)]$

(c) $[(\sim p \vee r) \wedge (p \vee \sim r)] \wedge [(\sim q \vee r) \wedge (q \vee \sim r)]$

(d) $[(\sim p \vee r) \wedge (p \vee \sim r)] \vee [(\sim q \vee r) \wedge (q \vee \sim r)]$

(e) $\sim [(\sim p \vee r) \wedge (p \vee \sim r)] \vee [(\sim q \vee r) \wedge (q \vee \sim r)]$

Problem 6 Proof: 10pts

For all integers a, b, m, n if m%d = a and n%d = b, does that mean
(m + n)%d = (a+b)?
If yes please prove; if no, please give counter example (assuming that d ∈ Z +)

Problem 7 Proof: 10pts

Please show that for any integer m and n, $m^2 – n^2$ is even if only if $m - n$ is even.

(a) Given any integer m and n, if $m^2 – n^2$ is even, then $m - n$ is even

(b) Given any integer m and n, if $m - n$ is even, then $m^2 – n^2$ is even

Problem 8 Proof: 10pts

Given a function f(x) = (x − 3) (x − 5) (x − 7) is odd for all integer x if only if x is even. Here if only if means p → q and q → p.
Please show

(a) f(x) is odd → x is even

(b) x is even → f(x) is odd