

MAT 115: Problem Set 3

Due: 11/04/2019

Instructions:

I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Score: /120

Bonus: /10

Total: /130

Problem 1 GCD (10 pts)

Use the Euclidean Algorithm to find the GCD of the following:

(a) 1001 and 544

(b) 3510 and 652

Problem 2 GCD (10 pts)

Find all common divisors of 51 and 288 using the Euclidean algorithms

Problem 3 Euler Function: (10 pts)

(a) Given $n = 162$, find $\varphi(n)$

(b) Given $n = 210$, find $\varphi(n)$

Problem 4 Venn Diagram: 20 pts

Prove or give a counterexample. Use a Venn diagram argument for the proof. For the counterexample, use a Venn diagram or use set specialization.

(a) If A , B , and C are subsets of U , then $(A - C) \cap (B - C) \cap (A - B) = \emptyset$.

(b) If A and B are subsets of U and if $A \subseteq B$, then $A \cap (U - B) = \emptyset$.

(c) If A , B , and C are subsets of U , and if $A \subseteq B$, then $A \cap (U - (B \cap C)) = \emptyset$.

(d) If A , B , and C are subsets of U , and if $(B \cap C) \subseteq A$, then $(A - B) \cap (A - C) = \emptyset$.

(e) If A and B are subsets of U and if $A \cap B = \emptyset$, then $A \times B = \emptyset$.

Problem 5: Subset: (3 * 5 = 15pts)

Answer the following about \in and \subseteq operators.

(a) Is $\{1, 2\} \in \{\{1, 2\}, \{3, 4\}\}$?

(b) Is $\{2\} \in \{1, 2, 3, 4\}$?

(c) Is $\{3\} \in \{\{1, 2\}, \{3\}, \{4\}\}$?

Problem 6 Induction: (15 points)

Prove by induction that $n^2 \leq 2^n$ for all $n \geq 4$.

Problem 7: Venn Diagram: (5+5) pts

For each of the following, draw a Venn diagram.

(a) $A \subseteq B, C \subseteq B, A \cap C = \emptyset$

(b) $A \supseteq C, B \cap C = \emptyset.$

Problem 8: Power sets: (10)pts

Use the characteristic function format to list the power set of the following sets. That is, describe each element of the power set as a vector of zeroes and ones.

(a) $\{1, 2, 3\}$

(b) $X \times Y$ where $X = \{a, b\}$ and $Y = \{x, y\}$.

Problem 9: Algebraic rules (10 points)

Prove that $Q - (P \cap R) = Q - (P \cap Q \cap R)$.

Problem 10 : Functions (10pts)

Let $A = \{1, 2, 3\}$ and $B = \{a, b, d\}$. Consider the following subsets of sets.

$\{(3, a), (2, b), (1, a)\},$
 $\{(1, a), (2, b), (1, d)\},$

$\{(1, a), (2, b), (3, c)\},$
 $\{(1, a), (2, b), (3, d), (1, b)\}.$

Which of them are relations on $A \times B$? Which of the are functional relations? Which of their inverses are functional relations?

Problem 11 : Bonus Problem (10 pts)

How many refinements are there of the partition $\{\{1, 3, 5\}, \{2, 6\}, \{4, 7, 8, 9\}\}$? Explain.