CS 495 & 540: Problem Set 4

Section: MW 10-11:50 am

Instructions:
1. I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

2.Aim for concise solutions, as it will save you time spent on write-ups, and also help you conceptualize the key idea of the problem.

First Name:

Last Name:

Group ID:

Score: /

Problem 1 Baye's Net Indepdence: D-Separation

Given the following graphs, please
(I) Verify and explain if the statement is true

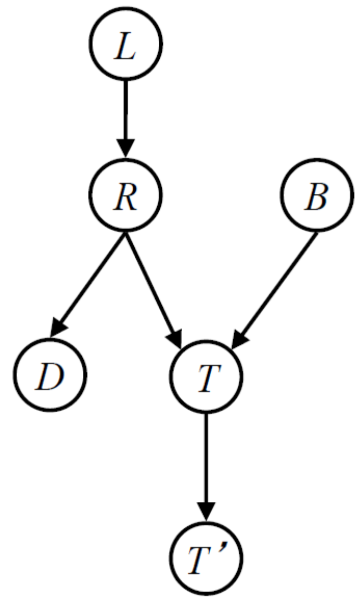
$$L \perp\!\!\!\perp T' | T$$

$$L \perp\!\!\!\perp B$$

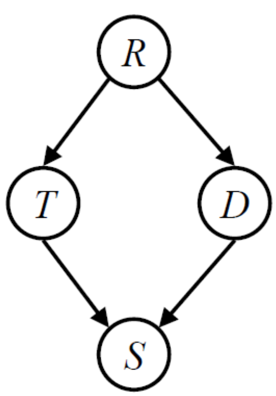
$$L \perp\!\!\!\perp B | T$$

$$L \perp\!\!\!\perp B | T'$$

$$L \perp\!\!\!\perp B | T, R$$



(II) Find independences in this graph (please consider both shade and no shade).



Problem 2 Bay's Net

We have a bag of three biased coins a, b, and c with probabilities of coming up heads of 20%, 60%, and 80%, respectively. One coin is drawn randomly from the bag (with equal likelihood of drawing each of the three coins), and then the coin is flipped three times to generate the outcomes X1, X2, and X3.

a. Draw the Bayesian network corresponding to this setup and define the necessary CPTs.

b. Calculate which coin was most likely to have been drawn from the bag if the observed flips come out heads twice and tails once.

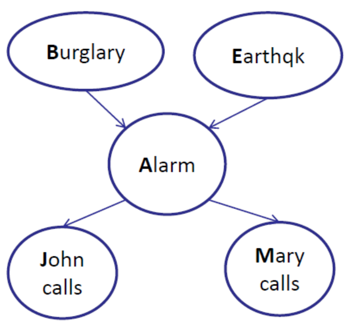
Problem 3 Independence

Consider the Bayesian network given at the bottom of the page: (a) If no evidence is observed, are Burglary and Earthquake independent? Prove this from the numerical semantics and from the topological semantics.

(b) If we observe Alarm = true , are Burglary and Earthquake independent? Justify your answer by calculating whether the probabilities involved satisfy the definition of conditional independence

B	P(B)
+b	0.001
-b	0.999

E	P(E)
+e	0.002
-e	0.998



A	J	P(J|A)
+a	+j	0.9
+a	-j	0.1
-a	+j	0.05
-a	-j	0.95

A	M	P(M|A)
+a	+m	0.7
+a	-m	0.3
-a	+m	0.01
-a	-m	0.99

(c) Compute P(+a,-b,+e,-j, +m), P(-a,-b,-e,+j, +m) and P(+a,-b,+e,+j,+m).

Problem 4 3-SAT and Bayesian Network

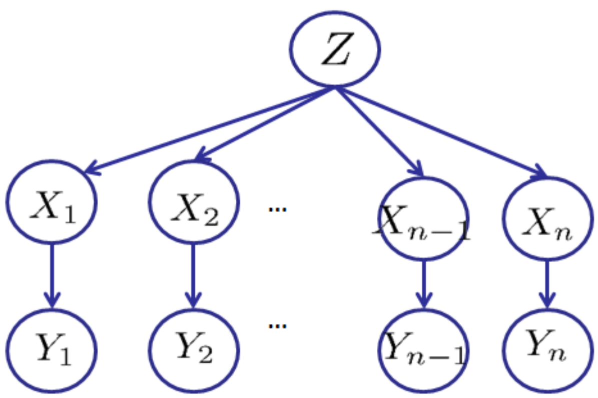
Investigate the complexity of exact inference in general Bayesian networks:
(a) Prove that any 3-SAT problem can be reduced to exact inference in a Bayesian network constructed to represent the particular problem and hence that exact inference is NP-hard. (Hint: Consider a network with one variable for each proposition symbol, one for each clause, and one for the conjunction of clauses.)

(b) The problem of counting the number of satisfying assignments for a 3-SAT problem is #P-complete. Show that exact inference is at least as hard as this.

Problem 5 Inference by Enumeration/Elimination

(I) Given an initial probability distributions on various variables that can lead to a joint probability distribution, you should be able to compute (approximate) the computation cost when comparing the enumeration approach and the elimination approach (as seen in lecture in Baye's Nets: Inference).

(II) Given the following Baye's Inference graph, for the query $P(X_n|y_1,,y_n)$ work through the following two different orderings as done in previous slide: $Z, X_1,, X_{n-1}$ and $X_1,, X_{n-1}, Z$. What is the size of the maximum factor generated for each of the orderings?



Problem 6 Digit Recognition

As learned from class, we know about the digit recognition. The written number is recognized as a specific number because that specific number has the highest probability resembling the written number, based on a set of training data. Now, if you are asked to write some software for signature recognition for the banking system, what would you do? Please briefly describe our algorithm and the possible drawbacks. And how you would fix the drawbacks. It might be to your advantage if you study some of the smoothing techniques.