CS 538: Problem Set 2

Section: MW 2-3:15 pm

Total: 60pts Due: 03/27/2017

Instructions:
1. I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

2. This assignment contains two parts. You are allowed to work on the homework in a group (you can stick to your notes-taking group or form a different one). No late assignment is accepted. Identical solutions (same wording, paragraph, code), turned in by different groups (persons), will be considered cheating.

3. Full credit will be given only to the correct solution which is described clearly. Convoluted and obtuse descriptions might receive low marks, even when they are correct. Also, aim for concise solutions, as it will save you time spent on write-ups, and also help you conceptualize the key idea of the problem.

First Name:

Last Name:

Group ID:

Score: **/ 60**

Problem 1 Min $\leftrightarrow$ Max: Find Dual: 20pts

Please convert the following problems from min (max) LP to max (min) LP
(A) minimize: $5x_1 + 7x_2 + 6x_3 + 2x_4$
subject to: $2x_1 + x_2 + x_3 + 3x_4 > 5$
$x_1 + 3x_2 + x_3 + 2x_4 = 5$
$x_1, x_2 \geq 0$
x_3, x_4 are unconstrained

(B) maximize: $3x_1 + 6x_2 + 7x_3 + 7x_4$
subject to: $5x_1 + 2x_2 + x_3 + 2x_4 < 20$
$x_1 + 3x_2 + 2x_3 + 2x_4 < 30$
$x_1, x_2, x_3, x_4 \geq 0$

Problem 2 LP: 15pts

In a facility location problem, there is a set of facilities and a set of cities, and our job is to choose a subset of facilities to open, and to connect every city to some one of the open facilities. There is a nonnegative cost f_j for opening facility j, and a nonnegative connection cost $c_{i,j}$ for connecting city i to facility j. Given these as input, we look for a solution that minimizes the total cost. Formulate this facility location problem as an integer programming problem and find its LP relaxation.

Problem 3 Maximum Covering: 10 + 15pts

Consider the following maximum covering problem. Given a graph G and a given number k, find a subset of k vertices that touches the maximum number of edges. Let $opt(G; k)$ be the optimal number of edges touched in G by a set of at most k vertices.
(a) Please design an algorithm for obtaining $opt(G, k)$

(b) Design an integer programming formulation for the problem, and then find a randomized rounding procedure for the corresponding linear programming relaxation, such that for given G and k, it identifies a set of at most $2k$ vertices that touches at least $c \cdot opt(G, k)$ edges, for some positive constant c.