CS 538: Problem Set 3

Section: MW 2-3:15 pm

Total: 60pts Due: 04/24/2017

Instructions:
1. I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

2. Full credit will be given only to the correct solution which is described clearly. Convoluted and obtuse descriptions might receive low marks, even when they are correct. Also, aim for concise solutions, as it will save you time spent on write-ups, and also help you conceptualize the key idea of the problem.

First Name:

Last Name:

Group ID:

Score: **/ 80**

Problem 1 Chernoff Bound: Q.10.1 on P. 267: 20pts

Theorem 1. *Let $X_1, \cdots, X_m$ be indepdent and identically distributed indicator random variables, with $\mu = \mathbb{E}[X_i]$. If $m \geq (3\ln(2/\delta))/(\epsilon^2\mu)$ then*

$$Pr\Big(|\frac{1}{m}\sum_{i=1}^{m} X_i - \mu| \geq \epsilon\mu\Big) \leq \delta$$

Please prove the above theorem by applying Chernoff bound.

Problem 2 Application: DNF Counting 4*5 pts

In class we talk about two algorithms (see chapter 10 in the textbook) for estimating the number of solutions for a given disjunctive normal form F. Let us call the Naive approach Alg-I (sampling from the solution space) and the one that cleverly samples from the TRUE solutions space as Alg-II. Let us assume that F contains t clauses and n variables. Please answer the following:
(a) Why is Alg-I not a good sampling approach

(b) In Alg-II, why is $|S|/|U| \geq 1/t$? In what scenario we will have $|S|/|U| = 1/t$? And in what scenario we have $|S|/|U| = 1$?

(c) If F is a k-DNF, what is the value $|SC_i|/|U|$ Is it a fixed number for all i?

(d) What is the major difference in sample space between Alg-I and Alg-II and why Alg-II is better?

Problem 3 Application: DNF Alg-I: Q.10.4 on P. 267: 20pts

Suppose we have a class of instances of the DNF satifiability problem, each with $\alpha(n)$ satisfying truth assignments for some polynomial α. Suppose we apply the naive appraoch of sampling assginments and checking wether they satisfy the formula. Show that, after sampling $2^{n/2}$ assginments, the probability of finding even a single satifying assignment for a given instance is exponenitally samll in n. [Hint: Union bound]

Problem 4 FPRAS: : Q.10.3 on P. 267: 20pts

Show that the following alternative definition is equivalent to the definition of an FPRAS given in the chapter: A *fully polynomial randomized approximation scheme (FPRAS)* for a problem is a randomized algorithm for which given an input x and any parameter ϵ with $0 < \epsilon < 1$ the algorithm outputs and $(\epsilon, 1/4)$-approximation in time that is polynomial in $1/\epsilon$ and the size of the input x (Hint: To boost the probability of success from $3/4$ to $1 - \delta$ consider the median of several independent runs of the algorithm. Why is the median a better choice than the mean?)

Problem 5 Practice 1: Q.10.6 on P. 267: No need to turn in

In question 10.6 in the textbook, it was mentioned that we can apply a strategy to throw away edges. How did you determine $k(n)$ and how effective is this approach?