MAT 115: Finite Math for Computer Science
Problem Set 1

Due: 02/06/2017

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Group ID:

Score: /110

Problem 1 Truth Table : 10pts

Make a truth table for $((p \to q) \wedge (\sim p \oplus r)) \wedge \sim (q \vee \sim r)$

Problem 2 Conditional Statements: Q.27 on P.15: 2+2+2pts

State the converse, contrapositive and inverse of these conditional statements
(a) If it snows today, I will ski tomorrow.

(b) I come to class whenever there is going to be a quiz

(c) A positive integer is a prime only if it has no divisors other than 1 and itself

Problem 3 Logic Reasoning: Q.40, P.16: 10pts

Explain, without using a truth table, why $(p \vee \sim q) \wedge (q \vee \sim r) \wedge (r \vee \sim p)$ is true when p, q, and r have the same truth value and it is false otherwise.

Problem 4 Circuit Design: 10 + 4 pts

1) Please design the circuit for the XOR gate by using AND, NOT and OR gate (Please try to come up with one that is different from what is given in class).

2) Please verify your circut with the four possible inputs by showing the ouputs after each gate and then the final output.

Problem 5 Circuit Design: Q.43, P.24 : 10pts

Construct a combinatorial circuit using NOT gates, OR gates and AND gates that produces the output $((\sim p \vee \sim r) \wedge \sim q) \vee (\sim p \wedge (q \vee r))$ from input bits p, q, and r.

Problem 6 Circuit: 10pts

Please explain why the full adder has OR at the end, instead of another half adder. Also, is it OK to replace that OR gate with XOR gate? Why?

Problem 7 Proposition Equivalence Q.10 & Q.12 on P.35: 8+8 pts

(a) Show that $((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology (always evaluates to truth) by using a truth table

(a) Show that $((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology (always evaluates to truth) by using algebraic rules (proposition equivalence rules)

Problem 8 Proposition Equivalence Q.20 & Q.21 on P.35 8+8 pts

(a) Show that $\sim (p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.

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Problem 9 Research: 10pts

We have been learning logic operators and those boolean functions. One type of well-studied boolean functions is 3-SAT. Please simply describe how a 3-SAT function is defined and its possible applications (maybe be good to read about NP-complete if you are interested in computational complexity).

Problem 10 Representing Function: 8pts

Given $f : \{0,1\}^3 \to \{0,1\}$, we can easily interpret it as $f(p,q,r) = s$ where $p,q,r \in \{0,1\}$ and $s \in \{0,1\}$. If we have $f(0,0,0) = 1, f(0,1,0) = 1, f(1,0,0) = 0, f(1,1,0) = 1, f(0,0,1) = 1, f(0,1,1) = 1, f(1,0,1) = 0$ and $f(1,1,1) = 1$. Please derive the boolean function f.