

MAT 115: Finite Math for Computer Science

Problem Set 2

Due: 02/20/2017

Instructions:

I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Score: /110

Problem 1 Basic Base Change: 5 + 5 pts

Convert the following numbers into a different base:

(A) 4ECB3 (base 16) to a base 10 number

(B) 367 (base 10) to a base 7 number

Problem 2 Simple Proof: Q.17 on P.92: 10pts

Given $n \in \mathbb{Z}$, show that if $n^3 + 5$ is odd, then n is even using

(a) a proof by contraposition

(b) a proof by contradiction

Problem 3 Predicate Logic: 10pts

$D = \{1, 3, 4, 5, 9, 121, 169, 196, 225, 289, 441\}$, $S(x) = (\sqrt{x} \in \mathbb{Z} \wedge \sqrt{x} + 2 \in \mathbb{P})$ where \mathbb{P} is the set of prime numbers. Let truth set $T = \{x \in D \mid S(x)\}$. Please show the elements inside the set T .

Problem 4 Proof: 10pts

Please show that for any integer m and n , $m^3 - n^3$ is even **if only if** $m - n$ is even.

(a) Given any integer m and n , if $m^3 - n^3$ is even, then $m - n$ is even

(b) Given any integer m and n , if $m - n$ is even, then $m^3 - n^3$ is even

Problem 5 Predicate: Q.16 on P. 53: 12pts

Determine the truth value of each of these statements, given $x \in \mathbb{R}$. Furthermore, if the truth value is false, please show a counter example.

(a) $\exists x(x^2 = 2)$

(b) $\exists x(x^2 = -1)$

(c) $\forall x(x^2 + 1 \geq 2)$

(d) $\forall x(x^2 \neq x)$

Problem 6 Quantifier: Q.50 on P.56: 10pts

Show that $\forall x P(x) \vee \forall x Q(x)$ and $\forall x (P(x) \vee Q(x))$ are not logically equivalent.

Problem 7 Nested Quantifier: Q.26, Q28 on P.67: 3*5pts

Let $Q(x)$ be the statement $x + y = x - y$, given $x, y, z \in \mathbb{Z}$. what are the truth value of the following expressions? And why?

(a) $\exists x \exists y Q(x, y)$

(b) $\exists y \forall x Q(x, y)$

(c) $\forall x \forall y Q(x, y)$

(d) $\forall x \forall y \exists z (z = (x + y)/2)$

(e) same as (d) but now $x, y, z \in \mathbb{R}$

Problem 8 Simple Induction: 2+3+5 pts

Please use induction proof method to show that $\sum_{i=1}^n i^2 + 3*i$ has a closed form as $\frac{n(n+1)(n+5)}{3}$

Base case:

Hypothesis:

Induction:

Problem 9 Proof: Triangle Inequality Q.7on P. 108: 10pts

Prove the **triangle inequality**, which states that if x and y are real numbers, then $|x|+|y| \geq |x + y|$ (where $|x|$ represents the absolute value of x).

Problem 10 Mersenne Number: 5 + 3 + 5 pts

We talked about Mersenne Number and Mersenne Prime Number in class. Please reserach on the internet and answer the following:

(a) List 2 applications of prime numbers in computer science

(b) What is the current biggest prime number? Is it a Mersenne Prime?

(c) Based the exercise 2.21 on Lo-22 (in the supplementary text) and the answer you have in (b), what is the current largest perfect number