MAT 115: Finite Math for Computer Science
Problem Set 4

Out: 03/27/2017 Due: 04/03/2017

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Score: **/85**

Problem 1 Relation: Q.1, P.615: 10pts

Which of these relations on $\{0, 1, 2, 3\}$ are equivalence relations? Determine the properties of an equivalence relation that the others lack.
a) $\{(0, 0), (1, 1), (2, 2), (3, 3)\}$
b) $\{(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)\}$
c) $\{(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$
d) $\{(0, 0), (1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$
e) $\{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)\}$

Problem 2 Relation: Q16, P.615: 15pts

Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $a + d = b + c$. Show R is an equivalence relation

Reflexive:

Symmetric:

Transitive:

Problem 3 Relation: Q9, P.615: 5*4 = 20pts

Suppose that A is a nonempty set, and f is a function that has A as its domain. Let R be the relation on A consisting of all ordered pairs (x, y) such that $f(x) = f(y)$.
a) Show that R is an equivalence relation on A.
Reflexive:

Symmetric:

Transitive:

b) What are the equivalence classes of R?

Problem 4 Functions: (5 + 10 + 5)

(a) Let $A = \{a, b, c, d\}, B = \{1, 2, 3, 4, 5\}$ where function f is defined as
$f : A \to B$ such that $f(a) = 1, f(b) = 2, f(c) = 3, f(d) = 3$
(a-1) Is this function f Injective or Surjective or Bijective or None? Why?

(b) Let fucntion f be defined as $f : A \to B$ where $A, B.$ are sets of intergers.
(b-1) Please show that if f is injective, then $|A| \leq |B|$

(b-2) Let $A = \{1, 2, 3, 4\}$ and $B = \{5, 6, 7, 9, 8\}$. Please draw a mapping to deny the statement that if $|A| \leq |B|$ then f is injective.

Problem 5 Functions: Q.20, P.153: 5*4 =20 pts

Let $f : A \to A$ where $A = \{1, 2, 3, 4, 5\}$. Give an example of f such that f is a) one-to-one but not onto.

b) onto but not one-to-one.

c) both onto and one-to-one (but different from the identity function).

d) neither one-to-one nor onto.

Problem 6 Extra: You don't have to turn in the solutions

Let R be the relation on the set of all URLs (orWeb addresses) such that xRy if and only if the Web page at x is the same as the Web page at y. Show that R is an equivalence relation.(c) Where is the pea?

Problem 7 Extra: You don't have to turn in the solutions

Define integers $x \equiv y$ to be related if $d|(x-y)$. Show that $\equiv$ is an equivalence relation by defining a function M that xMy when $d|(x-y)$.

Problem 8 Extra: You don't have to turn in the solutions

Let $A = \{1,2,3,4\}$ and $R = \{(1,1),(2,2),(3,3),(4,4),(1,4),(4,1),(2,3),(3,2)\}$.
(a) Is R an equivalence relation? (need to verify those three properties)
(b) What are the equivalent classes (partitions) of A
(c) $A = \{1,2,3,4,5,6\}$ and it can be partitioned by R_2 such that the partitions are $P_1 = \{1,5\}, P_2 = \{2,4,6\}$ and $P_3 = \{3\}$. Please write out the relation R_2.