MAT 115: Finite Math for Computer Science
Problem Set 5

Out: 04/10/2017 Due: 04/17/2017

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Score: **/100**

Problem 1 Permutation: the length of the cycle 3+4+4+4 pts

All the permutations given below are in cycle form. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$
(a) Please compute $((1, 3), (2, 5, 4))^{300}$

(b) $f : A \to A$ is a permuation and $f = (3, 4, 5, 2, 1, 7, 6), g = (1, 4, 6, 3, 2, 5, 7))$.
(1) f in cycle form

(2) $(f \circ g)^{-1}$ in 2 line form

(3) What is the period of $(f \circ g)^{-1}$

Problem 2 Pigeonhole Concept (10pts)

As seen in class, we have a set $A = \{a_1, a_2, \cdots, a_t\}$ be a set containing t distinct positive integers. Suppose we expect to have $a_i + a_j + a_k = a_l + a_m + a_n$ occur under the modulo function N where (1) $1 \leq i, j, k, l, m, n \leq t$ and i, j, k are distinct numbers (2) l, m, n are distinct integers and (3) $(i, j, k) \neq (l, m, n)$. Please find the smallest postive number t when $N = 97$ [Hint: Translate via $C(t, 3)$].

Problem 3 Permutation Application: 5 + 5 + 5 pts

The clown is playing the pea and the cup trick at your birthday party with 1 pea and 5 cups. He places a pea under the third cup. He quickly interchanges the cups in the second and the third positions then the cups in the first and the third positions and then the cups in the second and the third positions. Finally he interchanges the second and the fourth cup. The **entire set** of interchanges is done a total of six times.

(a) Write one entire set of interchanges as a permutation in cycle form:

(b) Write one entire set of interchanges as a permutation in adjacency matrix form:

(c) Where is the pea?

Problem 4 Permutation Application: Q.23, P.414: 5 + 5 pts

(a) How many ways are there for 10 women and six men to stand in a line so that no two men stand next to each other?

(b) Same as (a) but now the women are labeled $\{A, B, C, D, E, F, G, H, I, J\}$ and the men are labeled $\{\alpha, \beta, \gamma, \delta, \eta, \sigma\}$

Problem 5 Pigeonhole Concept (10pts)

Let $t_1, t_2, \cdots t_n$ be n **distinct** integers. Show that either $n|t_k$ for some k or $n|(t_i - t_j)$ for some $i \neq j$ (Hint: classsify those $t_1, \cdots t_n$ numbers by a modulo n function).

Problem 6 Permutation: With and Without Repetition ($5 \times 3 = 15$ pts)

We are interested in forming 3 letter words using the letters in Massachusetts. For the purpose of the problem, a word is any **list** of letters. Please answer the following:
(a) How many words can be made with no repeated letters?

(b) How many words can be made with unlimited repetition allowed?

(c) How many words can be made if repeats are allowed but no letter can be used more than it appears in Massachusetts?

Problem 7 Permutation: No Repetition 1 + 4*6=25

We work with the ordinary alphabet of **28-letters** (A-Z plus τ, λ. Please solve the following:
(a) Define a 5-letter word to be any list of 5 letters that contains *at least* one of the vowels A, E, I, O and U. How many 5-letter words are there?

(b) We can solve (a) in one single step but we can do it in 6 steps. Here are the steps:
(b-1) How many 5-letter words with exactly 1 vowel

(b-2) How many 5-letter words with exactly 2 vowels

(b-3) How many 5-letter words with exactly 3 vowels

(b-4) How many 5-letter words with exactly 4 vowels

(b-5) How many 5-letter words with exactly 5 vowels

(b-6) Your sum from b-1 till b-5 is? The result should be equivalent to (a)

Problem 8 Permutation: With Repetition (No need to turn in solution)

We work with the ordinary alphabet of 26-letters. Please solve the following:
(a) Define a 5-letter word to be any **list** of 5 letters that contains *at least* one of the vowels A, E, I, O and U. How many 5-letter words are there?
(b) We can solve (a) in one single step but we can do it in 6 steps. Here are the steps:
(b-1) How many 5-letter words with exactly 1 vowel
(b-2) How many 5-letter words with exactly 2 vowels
(b-3) How many 5-letter words with exactly 3 vowels
(b-4) How many 5-letter words with exactly 4 vowels
(b-5) How many 5-letter words with exactly 5 vowels
(b-6) Your sum from b-1 till b-5 is? The result should be equivalent to (a)

Problem 9 Combinatorial (No need to turn in solution) Q.28, P. 414

A professor writes 40 discrete mathematics true/false questions. Of the statements in these questions, 17 are true. If the questions can be positioned in any order, how many different answer keys are possible?

Problem 10 Pigeon Holes (No need to turn in solution) Q.34, P. 406

Assuming that no one has more than 1,000,000 hairs on the head of any person and that the population of New York City was 8,008,278 in 2010, show there had to be at least nine people in NewYork City in 2010 with the same number of hairs on their heads.