MAT 115: Finite Math for Computer Science
Problem Set 3

Due: 03/27/2017

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Score: **/100**

Problem 1 Power set: 5 + 5 + 5 pts

Given a set $A = \{a, c, d, t\}, B = \{0, 2, 5\}, C = \{e, s, y, k\}$, please compute the following:
(a) A^2 in dictionary order

(b) $B^2 \times (A \times C)$ in lexigraphical order

(c) Power set of A

Problem 2 Combinatorial: 10 pts

Please show that $C(n, k) = C(n-1, k-1) + C(n-1, k-1)$

Problem 3 Power set : 5 + 5 pts

Suppose we are given two non-empty sets A and B. Compare the following paris of sets. Can they be equal? Is one a subset of the other? Are they of the same size?
(a) $\mathcal{P}(A \cup B)$ and $\mathcal{P}(A) \cup \mathcal{P}(B)$

(b) $\mathcal{P}(A \times B)$ and $\mathcal{P}(A) \times \mathcal{P}(B)$

Problem 4 Perfect Number: Q.18 on P. 272 : 5 + 10 pts

We calla positive interger prfect if it equals the sum of its positive divisors other than itself.
(a) Show that 6 and 28 are perfect

(b) Show that $2^{p-1}(2^p - 1)$ is a perfect number when $2^p - 1$ is a prime.

Problem 5 GCD: Q.32 and Q.33 on P. 273: 3 + 3 + 4 pts

Use the Euclidean algoirthm and LCM to find
(a) gcd(12345, 54321)

(b) gcd(1529, 14039)

(c) Show that if a, b are positive integers then $ab = gcd(a, b) \times lcm(a, b)$

Problem 6 Sets: Proof: 5 + 5 pts

Prove each statement directly from the definitions.
(a) If A , B, and C are subsets of U, thne $A \subseteq B$ and $A \subseteq C$ implies that $A \subseteq B \cap C$.

(b) If A , B, and C are subsets of U, thne $A \subseteq C$ and $B \subseteq C$ implies that $A \cup B \subseteq C$.

Problem 7 GCD: *$Bézout's$* Theorem P. 269: 10 pts

Using the Euclidean algorithm, find A and B such that $Am+Bn = gcd(m,n)$ where $m = 252$ and $n = 180$.[Hint: the solution is not unique]

Problem 8 Venn Diagram: Q.1.6 on SF-13: 3 + 3 + 4 pts

Each of the following statements about subsets of a set U is FALSE. Draw a Venn diagram to represent the situation being described. In each case case, show that the assertion is false by specializing the sets.

(a) $\forall A, B$, and C, if $A \cap C \subseteq B \cap C$ and $A \cup C \subseteq B \cup C$ then $A = B$

(b) $\forall A, B$, and C $(A - B) \cap (C - B) = A - (B \cup C)$

(c) $\forall A, B$, and C, $(A - B) - C = A - (B - C)$

Problem 9 Venn Diagram: Q.34, Q40 on P. 137: 5 + 5 pts

The symmetric difference of A and B, denoted by A $\oplus$ B, is the set containing those elements in either A or B, but not in both A and B.
(a) Draw a Venn Diagram for the symmetric difference of the sets A and B.

(b) Determine where the symmetric difference is associative; that is if A, B and C are sets does it follow that $A \oplus (B \oplus C) = (A \oplus B) \oplus C$