MAT 115: Exam 1

Section: MW 10-11:50 am

Date: 03/01/2018

Instructions:
You have **100** minutes for this exam. There are 10 + 1 problems. The total score is **100** plus extra 10 points from a bonus problem. Please work on those in which you have confidence. If you finish the exam early, please just leave your exam on the desk and I will collect it. I leave plenty of space on each page for your computation. One extra blank sheet is also attached at the end of this exam if you need more computation space. If your answer is incorrect but you show the computation process, then partial credits will be given. Please use your time and space wisely.

First Name:

Last Name:

Score: **/100 +** **/10**

Problem 1 Truth Table (10 pts)

Make a truth table for $f = \sim (q \vee (\sim r \wedge q)) \wedge \sim ((\sim p \vee \sim r) \vee (\sim p \oplus r))$.

p	q	r	**f**
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Problem 2 Algebraic Rules and Boolean Functions (10pts)

Let p, q and r be boolean variables and let $f_1 := ((\sim p \vee q) \wedge (p \vee \sim r)) \wedge (\sim p \vee \sim q)$ and function $f_2 := \sim (p \vee r)$. Is f_1 equal to f_2? If yes, please **prove** it. If not, please simplify f_1 to its simplest form. Please use algebraic rules. If you are stuck, you can try truth table but you would only get half of the credits if your prove by truth table.

Problem 3 Base change + two's complement : 10pts

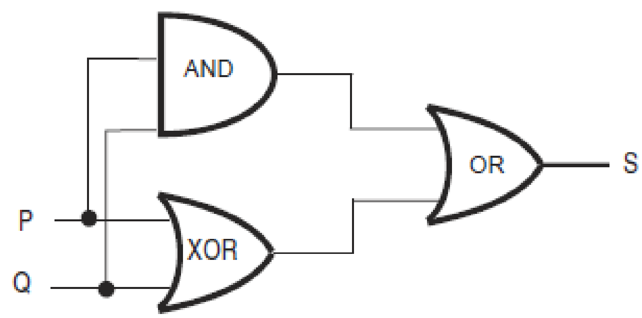
Express the 8-bit two's complement of the hexadecimal (base 16) number AB_{16} in base 5.

Problem 4 Circuit Design: 10pts

Construct a combinatorial circuit using NOT gates, OR gates and AND gates that produces the output $((\sim p \vee \sim r) \wedge q) \oplus (\sim p \wedge (q \vee r))$ from input bits p, q, and r. [Hint: You must express XOR gate in terms of NOT, OR and AND gates also; if you do not remember how to implement XOR gate, you can simply just use XOR gate in the circuit but partial credits will be deducted.]

Problem 5 Circuit Design: 10 pts (5+5)

Please derive the truth table for the following circuit and conclude what basic logic gate is equivalent to this circuit.



Problem 6 Propositional Logic: 5pts

Is $(p\wedge \sim q \wedge r) \wedge (\sim p \vee q \vee \sim r)\wedge \sim r$ a tautology, contradiction or neither? Please use algebraic rules.

Problem 7 Conditional Statements: 3*(3+2): 15 pts

With this propositional logic that if x is a pentagon, then x is a polygon, we can simply let P = x is a pentagon, Q = x is a polygon, then we know that $P \to Q$. With this statement, please show its conditional statements
(a) Contrapositve Statement. Is it a true statement and why?

(b) Converse Statement. Is it a true statement and why?

(c) Inverse statement. Is it a true statement and why?

Problem 8 Logic: Quantifiers (5+3+2 pts)

Is the following statement true or false? Explain why. If you disprove, you must provide the counter example.
(a) $\exists x \in D, (P(x) \wedge Q(x))$ is the same as $(\exists x \in D, P(X)) \wedge (\exists x \in D, Q(x))$

Let $Q(x)$ be the statement, given $x, y, z \in \mathbb{Z}$ and $x \neq 0$. what are the truth value of the following expressions? If true, find the corresponding values.

(a) $\exists x \exists y\ Q(x, y) := (x + y)/x = (x - y)$,

(b) $\exists Z \forall x \forall y\ Q(x, y) := ((x + y)/x)z = (x - y)z$,

Problem 9 Predicate Logic (10pts)

Let $D = \{1, 3, 4, 5, 9, 49, 64, 81, 121, 196, 225, 397\}$, $S(x) = (\sqrt{x} \in \mathbb{Z} \wedge ((\sqrt{x} - 1) \in \mathbb{P} \vee (\sqrt{x} + 2) \in \mathbb{P})) \vee (\sqrt{x + 3} \in \mathbb{Z} \wedge (\sqrt{x + 3})\%2 = 1)$. Let $T = \{x \in D | S(x)\}$. Please show the elements inside the set T.

Problem 10 Proof: 5 + 5pts

(a) For all odd integers n and m, please show $(3n+3)(m^2-n^2)$ is divisible by 12.

(b) Let $X = 1 + (3^1 - 2^1) + (3^2 - 2^2) + \cdots + (3^k - 2^k)$. Please find the closed form for X.

Problem 11 Bonus: Mersenne Prime: 2+8 pts

(a) What is a Mersenne Prime and what is a perfect number?

(b) Show that $N = 2^{n-1}(2^n - 1)$ is a perfect number when $2^n - 1$ is a Mersenne prime number.

Problem 12 Algebraic Rules Sheet

Theorem 2 (Algebraic rules for Boolean functions) *Each rule states that two different-looking Boolean functions are equal. That is, they look different but have the same table.*

Associative Rules:	$(p \wedge q) \wedge r = p \wedge (q \wedge r)$	$(p \vee q) \vee r = p \vee (q \vee r)$
Distributive Rules:	$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$
Idempotent Rules:	$p \wedge p = p$	$p \vee p = p$
Double Negation:	$\sim\sim p = p$	
DeMorgan's Rules:	$\sim(p \wedge q) = \sim p \vee \sim q$	$\sim(p \vee q) = \sim p \wedge \sim q$
Commutative Rules:	$p \wedge q = q \wedge p$	$p \vee q = q \vee p$
Absorption Rules:	$p \vee (p \wedge q) = p$	$p \wedge (p \vee q) = p$
Bound Rules:	$p \wedge 0 = 0 \qquad p \wedge 1 = p$	$p \vee 1 = 1 \qquad p \vee 0 = p$
Negation Rules:	$p \wedge (\sim p) = 0$	$p \vee (\sim p) = 1$

Problem 13 Scratch Paper

Do not detach the paper.

Problem 14 Scratch Paper

Do not detach the paper.