MAT 115: Exam 2 Version A

Section: MW 10-11:50 am

Date: 04/02/2018

Instructions:
You have **100** minutes for this exam. There are 10 + 2 problems. The total score is **100** plus extra 20 points from a bonus problem. Please work on those in which you have confidence. If you finish the exam early, please just leave your exam on the desk and I will collect it. I leave plenty of space on each page for your computation. One extra blank sheet is also attached at the end of this exam if you need more computation space. If your answer is incorrect but you show the computation process, then partial credits will be given. Please use your time and space wisely.

First Name:

Last Name:

Score: **/100 +** **/20**

Problem 1 Permutation Function

(a) In one line form, we have f=(1,2,4,5,3) and g = (4,5,1,2,3). What is $(g \circ f)^{-1}$ in cycle form ?

(b) Show that every permutation of 6 (i.e.$f : A \to A$ where $A = \{1, 2, 3, 4, 5, 6\}$), we have f^{60} being the identity permutation. What is f^{62}?

Problem 2 Relation

Let $S = \{1, 2, 3, \cdots 7\}$. For all $x, y \in S$, D is a relation of $X \times Y$ where $x < y$ and x divides y. How many pairs do we have in relation D? Explain.

Problem 3 Types of Functions: I

Let A and B be finite sets and $f : A \to B$. Prove/disprove the following:
(a) If f is injective, then $|A| \leq |B|$.

(b) If $|A| \leq |B|$, then f is injective.

Problem 4 Pigeon Holes

There are N students in the class. Their exam score ranges between 27 and 80. All possible scores are achieved by students except 45,67, 33, 35. What is the smallest N that guarantees that at least three students achieved the same score?

Problem 5 Equivalence Relation

Let $\mathbb{Z}$ be the set of integers. Define a relation R where $(a, b)R(c, d)$ and $a, b, c, d \in \mathbb{Z}$ except 0 if $ad = bc$. Please show R is an equivalence relation.
(a) Reflexive

(b) Symmetric

(c) Transitive

Problem 6 GCD

Find **all common divisors** of 252 and 198 using the Euclidean algorithms

Problem 7 Euler Function: pts

(a) Given $n = 165$, find $\phi(n)$

(b) Given $n = 231$, find $\phi(n)$

Problem 8 Subset

Answer the following about $\in$ and $\subseteq$ operators.
(a) Is $\{1, 2\} \in \{\{1, 2\}, \{3, 4\}\}$?

(b) Is $\{2\} \in \{1, 2, 3, 4\}$?

(c) Is $\{3\} \in \{\{1, 2\}, \{3\}, \{4\}\}$?

(d) Is $\{1, 2\} \subseteq \{1, 2, \{1, 2\}, \{3, 4\}\}$?

(e) Is $1 \in \{\{1\}, \{2\}, \{3\}\}$?

Problem 9 Set Algebraic Rules

(a) If A, B, C are subsets of U, then $(A - B) - C = (A - C) - B$

(b) If A, B, C, D are subsets of U, then $(A - B) - C - D = A - ((B \cup C) \cup D)$

Problem 10 Power Set

Here $\mathbb{P}$ here stands for power set.
(a) Let $S = \{1, 2, 3, 4\}$ and what is $\mathbb{P}(S)$?

(b)$\mathbb{P}(A \cap B) = \mathbb{P}(A) \cap \mathbb{P}(B)$? True or false? And why?

Problem 11 Bonus: Proof: Subset

(a) If A, B and C are subsets of U, then $A \subseteq B$ and $A \subseteq C$ implies that $A \subseteq (B \cap C$

(b) If A, B and C are subsets of U, then $B \subseteq C$ and $A \subseteq C$ implies that $A \cup B \subseteq C$

Problem 12 Bonus: Set Order

Let $A = \{x, y, z\}$ and $B = \{a, b, d\}$. Take the linear order on A and B to be alphabetical order. Compute the following int lexcographic orders.
(a) $B \times A$

(b) $A \times A$

Problem 13 Algebraic Rules Sheet

Associative:	$(P \cap Q) \cap R = P \cap (Q \cap R)$	$(P \cup Q) \cup R = P \cup (Q \cup R)$
Distributive:	$P \cap (Q \cup R) = (P \cap Q) \cup (P \cap R)$	$P \cup (Q \cap R) = (P \cup Q) \cap (P \cup R)$
Idempotent:	$P \cap P = P$	$P \cup P = P$
Double Negation:	$\sim\sim P = P$	
DeMorgan:	$\sim(P \cap Q) = \sim P \cup \sim Q$	$\sim(P \cup Q) = \sim P \cap \sim Q$
Absorption:	$P \cup (P \cap Q) = P$	$P \cap (P \cup Q) = P$
Commutative:	$P \cap Q = Q \cap P$	$P \cup Q = Q \cup P$

Problem 14 Scratch Paper

Do not detach the paper.

Problem 15 Scratch Paper

Do not detach the paper.