

MAT 115: Exam 3

Section: MW 10-11:50 am

Date: 04/25/2018

Instructions:

You have **100** minutes for this exam. The total score is **110** but we will put on the **100 scale**. That is, if you make 108, it means you make perfect score in this test plus extra 8 points. Please work on those in which you have confidence. If you finish the exam early, please just leave your exam on the desk and I will collect it. I leave plenty of space on each page for your computation. One extra blank sheet is also attached at the end of this exam if you need more computation space. If your answer is incorrect but you show the computation process, then partial credits will be given. Please use your time and space wisely.

First Name:

Last Name:

Score: /100

Problem 1 Permutations: 12 points

We have 6 kids to sit in a row. Let say the boys are A, B, C, D and the girls are E and F. Please compute the number of ways to seat the kids based on the following constraints

(a) How many way to sit those 6 distinct persons?

(b) C does not sit next to E and A does not sit next to D and B and F must sit together

(c) A must sit on the right of B, and E must sit at least 2 seats away from C (i.e. at least one 1 seat in between)

(d) B must sit next to C and D must also sit next to C and A does not want to sit next to E

Problem 2 Permutation + Combinatorics: (12 pts)

Please explain how many ways to put balls into bins based on the condition:

(a) 2 identical bins and 2 identical balls

(b) 2 different bins (A, B) and 2 identical balls

(c) 2 identical bins and 2 different balls (Red, White)

(d) 2 different bin (A, B) and 2 different balls (Red, White)

Problem 3 Permutation NO Repetition ($3 \times 6 = 18$ pts)

We work with the ordinary alphabet of 26-letters. Please solve the following:

(a) Define a 5-letter word to be any list of 5 letters that contains *at least* one of the vowels A, E, I, O and U. How many 5-letter words are there?

(b) We can solve (a) in one single step but we can do it in 6 steps. Here are the steps:

(b-1) How many 5-letter words with exactly 1 vowel

(b-2) How many 5-letter words with exactly 2 vowels

(b-3) How many 5-letter words with exactly 3 vowels

(b-4) How many 5-letter words with exactly 4 vowels

(b-5) How many 5-letter words with exactly 5 vowels

Problem 4 Permutation: (3+3+5 = 11 pts)

We are interested in forming 3 letter words using the letters in THELITTLEST.

(a) How many words can be made with no repeated letters?

(b) How many words can be made with unlimited repetition allowed?

(c) How many words can be made if repeats are allowed but no letter can be used more than it appears in THELITTLEST?

Problem 5 Probability: 10 pts

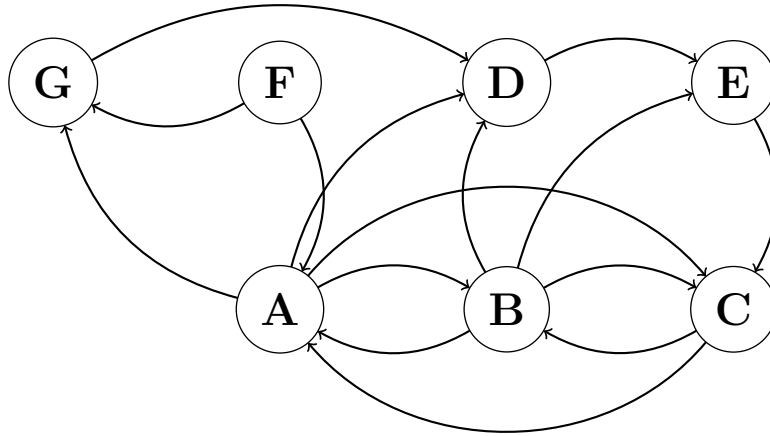
We are tossing 6 biased coins at the same time. The coins have the probability of 0.3 to get Head and 0.7 to get Tail.

(a) What is the probability of getting 5 Heads and 5 Tails?

(b) What is the probability of getting more Tails than Heads

Problem 6 Graph: Definition and Cycles: 6+5 pts

Suppose you are given the following **directed** graph $G = (V, E)$.



(a) Find 3 cycles within the graph. Cycle 1 contains 4 distinct vertices; cycle 2 contains 5 distinct vertices; Cycle 3 contains 6 distinct vertices.

(b) Is it possible to find a cycle of 7 distinct vertices? And why?

Problem 7 Induction Proof: 10 points

Please show that $\sum_{i=1}^n (i^2 + 3i) = \frac{(n)(n+1)(n+5)}{3}$

Base case:

Hypothesis

Induction

Problem 8 Stirling Number: 5+3+2 + 6 pts

For $n > k > 0$, the Stirling number of the 2nd kind is $S(n, k) = S(n - 1, k - 1) + k \times S(n - 1, k)$. A way to interpret it is how many ways to put n distinct objects into k identical bins while none of the bins should be empty.

(a) Please explain the rationale for this formula.

(b) By induction, show $S(n, 2) = 2^{n-1} - 1$

Base case:

Hypothesis:

Induction:

Problem 9 Probability + Counting (5 + 5 = 10 pts)

An urn A contains eleven labeled balls, labels 1,2, \dots , 5. An urn B contains six labeled balls, labels 1,2, \dots ,6.

(a) Two balls are drawn, one from A and one from B. What is the probability that the sum of the labels on the balls is 9?

(b) Two balls are drawn one after the other without replacement and the order matters from urn A. Then one ball is drawn from urn B. What is the probability that the sum of the labels on the balls is 8?

Problem 10 Scratch Paper

Do not detach the paper.

Problem 11 Scratch Paper

Do not detach the paper.