MAT 115: Final Exam

Section: MW 10-11:50 am

Date: 05/02/2018

Instructions:
You have **120** minutes for this exam. There are 14 problems. The total score is **140**. The exam will be on a total of 120 scale. For instance, if your score is 132, then means in addtion to a perfect score of 120/120, you also gain extra 12 points in the final. Recall that final has a higher weight on the grade. Please work on those in which you have confidence. If you finish the exam early, please just leave your exam on the desk and I will collect it. I leave plenty of space on each page for your computation. Extra blank sheets are attached at the end of this exam if you need more computation space. If your answer is incorrect but you show the computation process, then partial credits will be given. Please use your time and space wisely.

First Name:

Last Name:

Score: **/120**

Problem 1 Truth Table (10 pts)

Make a truth table for $f = \sim (q \vee (\sim r \wedge q)) \wedge \sim ((\sim p \vee \sim r) \wedge (\sim p \oplus r))$.

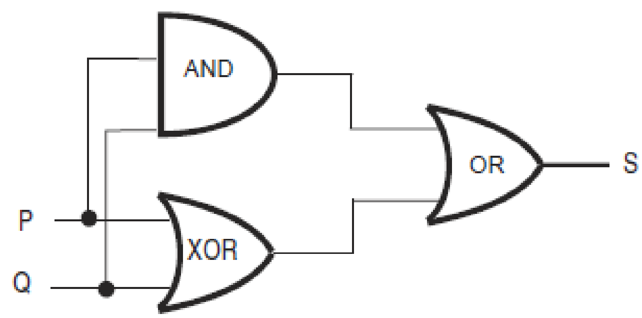
p	q	r	**f**
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Problem 2 Base change + two's complement : 10pts

Express the 8-bit two's complement of the hexadecimal (base 16) number EB_{16} in base 5.

Problem 3 Circuit Design: 10 pts (5+5)

Please derive the truth table for the following circuit and conclude what basic logic gate is equivalent to this circuit.



Problem 4 Propositional Logic: 5pts

Is $((p\wedge \sim q) \wedge (r \vee p)) \wedge ((\sim p \vee q) \vee (\sim r\wedge \sim p))\wedge \sim r$ a tautology, contradiction or neither? Please use algebraic rules.

Problem 5 Predicate Logic (10pts)

Let $D = \{1, 3, 4, 5, 9, 49, 64, 81, 121, 196, 225, 397\}$, $S(x) = (\sqrt{x} \in \mathbb{Z} \wedge ((\sqrt{x} - 1) \in \mathbb{P} \vee (\sqrt{x} + 2) \in \mathbb{P})) \vee (\sqrt{x + 3} \in \mathbb{Z} \wedge (\sqrt{x + 3})\%2 = 0)$. Let $T = \{x \in D | S(x)\}$. Please show the elements inside the set T.

Problem 6 Proof

(a) For all odd integers n and m, please show $(m^2 - n^2)$ is divisible by 4.

(b) For all odd integers n and m, please show $(3n + 3)(m^2 - n^2)$ is divisible by 24.

Problem 7 Permutation Function:

(a) In one line form, we have f=(1,4,3,5,2) and g = (5,1,4,2,3). What is $(g \circ f)^{-1}$ in cycle form ?

(b) (1) Show that every permutation of 6 (i.e.$f : A \to A$ where $A = \{1, 2, 3, 4\}$) we have f^{12} being the identity permutation. (2) What is f^{62}?

Problem 8 Types of Functions: I

Let A and B be finite sets and $f : A \to B$. Prove/disprove the following:
(a) If f is injective, then $|A| \leq |B|$.

(b) If $|A| \leq |B|$, then f is injective.

Problem 9 Equivalence Relation

Let $\mathbb{Z}$ be the set of integers. Define a relation R where $(a, b)R(c, d)$ and $a, b, c, d \in \mathbb{Z}$ except 0 if $ad = bc$. Please show R is an equivalence relation.

(a) Reflexive

(b) Symmetric

(c) Transitive

Problem 10 GCD

Find **greatest divisor** of 2610, and 189 using the Euclidean algorithms

Problem 11 Permutation

We are forming 3 letter words using the letters in THELITTLESTINUTICA.
(a) How many words can be made if repeats are allowed but no letter can be used more than it appears in THELITTLEST?

We have 8 kids to sit in a rowl. Let say the boys are A, B, C, D and the girls are E, F, G and H. Please compute the number of ways to seat the kids based on the following constraints
(a) A must sit on the right of B, and E must sit at least 2 seats away from C (i.e. at least one 1 seat in between), and F and H must sit together.

Problem 12 Stirling Number: 3+2 +5 pts

For $n > k > 0$, the Stirling number of the 2nd kind is $S(n,k) = S(n-1,k-1) + k \times S(n-1,k)$. By induction, show $S(n,2) = 2^{n-1} - 1$
Base case:

Hypothesis:

Induction:

Problem 13 Probability + Counting

An urn A contains eleven labeled balls, labels 1,2, $\cdots$, 10. An urn B contains six labeled balls, labels 1,2, $\cdots$,10.
(a) Two balls are drawn, one from A and one from B. What is the probability that the sum of the labels on the balls is 9?

(b) Two balls are drawn one after the other without replacement and the order matters from urn A. Then one ball is drawn from urn B. What is the probability that the sum of the labels on the balls is 8?

Problem 14 Mersenne Prime

(a) What is a Mersenne Prime and what is a perfect number?

(b) Show that $N = 2^{n-1}(2^n - 1)$ is a perfect number when $2^n - 1$ is a Mersenne prime number.

Problem 15 Algebraic Rules Sheet

Theorem 2 (Algebraic rules for Boolean functions) *Each rule states that two different-looking Boolean functions are equal. That is, they look different but have the same table.*

Associative Rules:	$(p \wedge q) \wedge r = p \wedge (q \wedge r)$	$(p \vee q) \vee r = p \vee (q \vee r)$
Distributive Rules:	$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$
Idempotent Rules:	$p \wedge p = p$	$p \vee p = p$
Double Negation:	$\sim\sim p = p$	
DeMorgan's Rules:	$\sim(p \wedge q) = \sim p \vee \sim q$	$\sim(p \vee q) = \sim p \wedge \sim q$
Commutative Rules:	$p \wedge q = q \wedge p$	$p \vee q = q \vee p$
Absorption Rules:	$p \vee (p \wedge q) = p$	$p \wedge (p \vee q) = p$
Bound Rules:	$p \wedge 0 = 0 \qquad p \wedge 1 = p$	$p \vee 1 = 1 \qquad p \vee 0 = p$
Negation Rules:	$p \wedge (\sim p) = 0$	$p \vee (\sim p) = 1$

Problem 16 Algebraic Rules Sheet

Associative:	$(P \cap Q) \cap R = P \cap (Q \cap R)$	$(P \cup Q) \cup R = P \cup (Q \cup R)$
Distributive:	$P \cap (Q \cup R) = (P \cap Q) \cup (P \cap R)$	$P \cup (Q \cap R) = (P \cup Q) \cap (P \cup R)$
Idempotent:	$P \cap P = P$	$P \cup P = P$
Double Negation:	$\sim\sim P = P$	
DeMorgan:	$\sim(P \cap Q) = \sim P \cup \sim Q$	$\sim(P \cup Q) = \sim P \cap \sim Q$
Absorption:	$P \cup (P \cap Q) = P$	$P \cap (P \cup Q) = P$
Commutative:	$P \cap Q = Q \cap P$	$P \cup Q = Q \cup P$

Problem 17 Scratch Paper

Do not detach the paper.

Problem 18 Scratch Paper

Do not detach the paper.

Problem 19 Scratch Paper

Do not detach the paper.

Problem 20 Scratch Paper

Do not detach the paper.