MAT 115: Finite Math for Computer Science
Problem Set 1

Due: 02/07/2018

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Group ID:

Score: **/100**

Problem 1 Truth Table : 10pts

Make a truth table for $((p \vee (\sim p \vee q)) \wedge \sim (q \wedge \sim r)$

Problem 2 Logic Reasoning: 10pts

Explain, without using a truth table, why $(p \vee \sim q) \wedge (q \vee \sim r) \wedge (r \vee \sim p)$ is true when p, q, and r have the same truth value and it is false otherwise. (Hint: you are dealing with 3 ANDs, and each clause must be 1 in order for the function to be evaluated to 1)

Problem 3 Representing Function: 10 pts

Given $f : \{0,1\}^3 \to \{0,1\}$, we can easily interpret it as $f(p,q,r) = s$ where $p,q,r \in \{0,1\}$ and $s \in \{0,1\}$. If we have $f(0,0,0) = 1, f(0,1,0) = 1, f(1,0,0) = 0, f(1,1,0) = 1, f(0,0,1) = 1, f(0,1,1) = 1, f(1,0,1) = 1$ and $f(1,1,1) = 0$. Please derive the boolean function f.

Problem 4 Algebraic Rules: 10 pts

Is the function $((\sim p \vee q) \wedge (p \vee \sim r)) \wedge (\sim p \vee \sim q)$ equal to the function $\sim (p \vee r)$?

Problem 5 Algebraic Rules: 10 pts

Show that $p \vee (p \wedge q) = p$ follows the distributive rule, idempoten rule and the absorption rule $p \wedge (p \vee q) = p$.

Problem 6 Circuit Design: 10 pts

Design a circuit that implements the Boolean function F where $F(P, Q, R) = 0$ if and only if $(P, Q, R) = (0, 0, 0)$ or $(P, Q, R) = (1, 1, 1)$. Hint: use representation function for the function

Problem 7 Circuit Design: 5 + 5 pts

1) Please design the circuit for the XOR gate by using AND, NOT and OR gate.

2) Please verify your circut with the four possible inputs by showing the ouputs after each gate and then the final output.

Problem 8 Circuit Design: 10 pts

You have a grumpy landlord and he believes in majority vote. Let say each light in the house is controlled by three switches. The light is only on when at least two of the switches are in the same position. Your electrician friend came and said that if you can design the circuit for him to follow such that it matches your landlord's strange requirement then he will wire for free and get you some beer. Otherwise, you have to buy him a lobster dinner at Red Lobster. And your landlord said he will reduce the rent by 200 if you can design the circuit. Wow, that is a lot of saving in a long term. Please draw the circuit to save yourself some money and show them (landlord and electrician friend) you are a serious and smart guy. [Hint: In order to show you are smart, you intend to give them the shortest circuit (least amount of gates used) you can come up with]

Problem 9 Base change + two's complement : 10pts

In octal (base 8), the 12-bit two's complement of the hexadecimal (base 16) number $2AF_{16}$ is? Please show the computation process. Hint: base 16 $\rightarrow$ base 10 $\rightarrow$ base 2 $\rightarrow$ two's complement of a 12-bit binary string $\rightarrow$ base 10 $\rightarrow$ base 8

Problem 10 Circuit: 10pts

Please explain why the full adder has OR at the end, instead of another half adder. Also, is it OK to replace that OR gate with XOR gate? Why?

Problem 11 Additional Problems

You do not need to turn in the following problems. But you are encourage to practice those problems. BF: 1.9 - 1.15; 2.5 - 2.15