MAT 115: Finite Math for Computer Science
Problem Set 4

Due: 04/02/2018

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely. Each problem is worth 10 points while each bonus problem is worth 5 points.

First Name:

Last Name:

Group ID:

Score: /

Problem 1 Permutation Function

Given the following permutation, please find:
(a) (1,5,7,8)(2,3)(6), please find its two-line form and its inverse in cycle form

(b) (5,4,3,2,1), which is in one-line form. Find its 2-line form and inverse in cycle form

Problem 2 Permutation Function

(a) Compute $((1,3)(2,5,4))^{300}$

(b) Show that every permutation of 5 (i.e.$f : A \to A$ where $A = \{1,2,3,4,5\}$), we have f^{60}being the identity permutation. What is f^{61}?

Problem 3 Permutation Function

(a)In one line form, we have f=(1,2,4,5,3) and g = (4,5,3,1,2). What is $f \circ g$?

(b) In one line form, we have f=(1,2,4,5,3) and g = (4,5,3,1,2). What is $g \circ f$?

(c) In one line form, we have f=(1,2,4,5,3) and g = (4,5,3,1,2). What is $g \circ (f \circ g^{-1})$?

Problem 4 Relation and Function

Find all relations on $\{a, b\} \times \{x, y\}$ that are not functional

Problem 5 Relation

Let $S = \{1, 2, 3, \cdots 10\}$. For all $x, y \in S$, D is a relation of $X \times Y$ where $x < y$ and x divides y. How many pairs do we have in relation D? Explain.

Problem 6 Function

Let $S = \{f | f : A \to B$, f is injective $\}$. In each case find $|S|$
(a) $|A| = 3, |B| = 3$

(a) $|A| = 3, |B| = 5$

(a) $|A| = m, |B| = n, m < n$

Problem 7 Types of Functions: I

Let A and B be finite sets and $f : A \to B$. Prove/disprove the following:
(a) If f is injective, then $|A| \leq |B|$.

(b) If $|A| \leq |B|$, then f is injective.

Problem 8 Types of Functions: II

Let A and B be finite sets and $f : A \to B$. Prove/disprove the following:
(a) If f is surjective, then $|A| \geq |B|$.

(b) If $|A| \geq |B|$, then f is surjective.

Problem 9 Equivalence Relation

Let $\mathbb{Z}$ be the set of integers. Define a relation R where $(x, y) \in R$ and $x, y \in \mathbb{Z}$ that 5 divides $x^2 - y^2$. Please show R is an equivalence relation.
(a) Reflexive

(b) Symmetric

(c) Transitive

Problem 10 Pigeon Holes

There are N students in the class. Their exam score ranges between 27 and 99. All possible scores are achieved by students except 45,67, 33. What is the smallest N that guarantees that at least three students achieved the same score?

Problem 11 Bous: Pigeon Holes

Let $t_1, \cdots, t_k$ be n different integers. Show that either n divides t_k for some k or n divides $(t_i - t_j)$ for some $i \neq j$

Problem 12 Bonus: Equivalence Relation

Let $\mathbb{Z}$ be the set of integers. Define a relation R where $(a, b)R(c, d)$ and $a, b, c, d \in \mathbb{Z}$ except 0 if $ad = bc$. Please show R is an equivalence relation.

(a) Reflexive

(b) Symmetric

(c) Transitive