MAT 115: Finite Math for Computer Science
Problem Set 5

Out: 04/11/2018 Due: 04/18/2018

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Group ID:

Score: **/137 + /20**

Problem 1 Permutations: 20 points

We have 7 kids to sit in a rowl. Let say the boys are A, B, C, D and the girls are E, F and G. Please compute the number of ways to seat the kids based on the following constraints
(a) Each girl must sit between boys

(b) C does not sit next to E and A does not sit next to G and B and F must sit together

(c) A must sit on the right of B, and E must sit at least 2 seats away from G (i.e. at least one 1 seat in between)

(d) B must sit next to C and D must also sit next to C and A does not want to sit next to G

Problem 2 Combinatorics: 10 points

You are controlling a robot. The task is given that your robot is set at coordinate $(3, 6)$ and your robot has to move to coordinate (10,14). Let say each time your robot can only move up by 1 in Y axis or right by one in the X axis. Let us assume each move is of the same cost, therefore, all possible paths are of the same total cost. Please show how many paths there are for your robot to move from

(a) $(3, 6)$ to $(10, 14)$

(b) $(3, 6)$ to $(10, 14)$ but must pass $(6, 8)$, $(7, 9)$ in the path.

Problem 3 Permutation + Combinatorics: (20 pts)

Please explain how many ways to put balls into bins based on the condition:
(a) 4 identical bins and 4 identical balls

(b) 4 different bins (A, B, C, D) and 4 identical balls

(c) 4 identical bins and 4 different balls (Red, White, Green, Purple)

(d) 4 different bin (A, B, C, D) and 4 different balls (Red, White, Green, Purple)

Problem 4 Permutation: With Partial Repetition (3×7 = 21 pts)

We work with the ordinary alphabet of 26-letters and only 2 of them, let say $\{X, Y\}$, are allowed to repeat. Please solve the following:
(a) Define a 5-letter word to be any list of 5 letters that contains *at least* one of the vowels A, E, I, O and U. How many 5-letter words are there?

(b) We can solve (a) in one single step but we can do it in 6 steps. Here are the steps:
(b-1) How many 5-letter words with exactly 1 vowel

(b-2) How many 5-letter words with exactly 2 vowels

(b-3) How many 5-letter words with exactly 3 vowels

(b-4) How many 5-letter words with exactly 4 vowels

(b-5) How many 5-letter words with exactly 5 vowels

(b-6) Your sum from b-1 till b-5 is? The result should be equivalent to (a)

Problem 5 Permutation: With NO Repetition (3×7 = 21 pts)

We work with the ordinary alphabet of 26-letters. Please solve the following:
(a) Define a 5-letter word to be any list of 5 letters that contains *at least* one of the vowels A, E, I, O and U. How many 5-letter words are there?

(b) We can solve (a) in one single step but we can do it in 6 steps. Here are the steps:
(b-1) How many 5-letter words with exactly 1 vowel

(b-2) How many 5-letter words with exactly 2 vowels

(b-3) How many 5-letter words with exactly 3 vowels

(b-4) How many 5-letter words with exactly 4 vowels

(b-5) How many 5-letter words with exactly 5 vowels

(b-6) Your sum from b-1 till b-5 is? The result should be equivalent to (a)

Problem 6 Permutation: With and Without Repetition (5×3 = 15 pts)

We are interested in forming 3 letter words using the letters in THELITTLESTGUY.
(a) How many words can be made with no repeated letters?

(b) How many words can be made with unlimited repetition allowed?

(c) How many words can be made if repeats are allowed but no letter can be used more than it appears in THELITTLESTGUY?

Problem 7 Probability: 20 pts

We are tossing 10 biased coins at the same time. The coins have the probability of 0.4 to get Head and 0.6 to get Tail.
(a) What is the probability of getting 10 Heads?

(b) What is the probabiliyt of getting 4 Heads and 6 Tails?

(c) What is the probability of getting more Tails then Heads

(d) What is the probability of getting even numbers of Tails

(e) What is the probability of getting odd numbers of Tails? Does your answer here along with answer in (d) sum up to 1?

Problem 8 Induction Proof: 10 points

Please show that $\sum_{i=1}^{n}(i^2 + 2i) = \frac{(n)(n+1)(n+2)}{3}$
Base case:

Hypothesis

Induction

Problem 9 Bonus: Permutation: Logic Flaw: 10 points

In class, we discussed an interesting scenario that we have 4 different spots and we need to fill them with 2 colors while each color occupy 2 spots. And let assume we have 3 colors to choose from. We know the right way to do is $C(3,2) \times P(4,2)$. But somehow some logic like, OK, let me pick 2 spots, then pick 1 color for those two boxes and then pick another color to fill the rest 2 boxes, the computation for that is $C(4,2) \times C(3,1) \times C(2,1)$. This is an overcount case. Can you explain where the overcount happens?

Problem 10 Bonus: Induction Proof: 10 points

Please show that $\sum_{i=1}^{n} i^2 = \frac{(n)(n+1)(2n+1)}{6}$
Base case:

Hypothesis

Induction