

MAT 115: Finite Math for Computer Science
Problem Set 6

Out: 04/20/2018 Due:

Instructions:

I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely.

First Name:

Last Name:

Group ID:

Score: /95

Problem 1 Stirling Number: 5+ 10 + 10 pts

For $n > k > 0$, the Stirling number of the 2nd kind is $S(n, k) = S(n-1, k-1) + k \times S(n-1, k)$. A way to interpret it is how many ways to put n distinct objects into k identical bins while none of the bins should be empty.

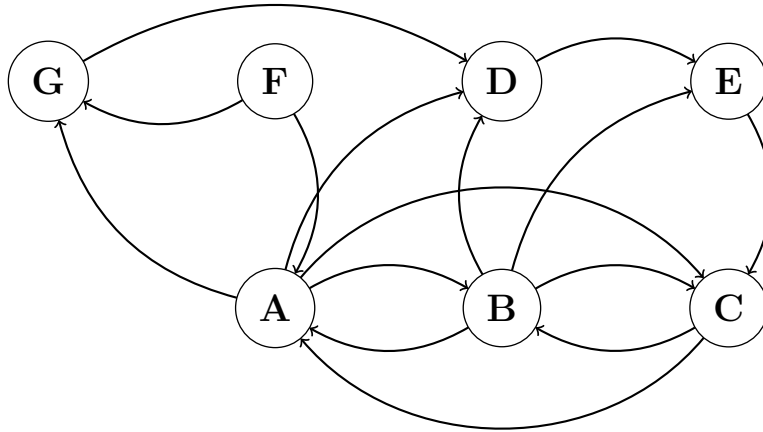
(a) This recursive formula is based on the the situation of tossing the very last object in to the bin. Please explain the rationale for this formula.

(b) Please find the close form for $S(n, 3)$ provided you know that $S(n, 2) = 2^{n-1} - 1$, $S(n, k) = S(n-1, k-1) + k \times S(n-1, k)$. (Hint: recursive calls add up the exponents)

(c) Please prove (c) using Induction

Problem 2 Graph: Definition and Cycles: $3 \times 3 + 5 + 10$ pts

Suppose you are given the following **directed** graph $G = (V, E)$.



(a) Find 3 cycles within the graph. Cycle 1 contains 4 distinct vertices; cycle 2 contains 5 distinct vertices; Cycle 3 contains 6 distinct vertices.

(b) Is it possible to find a cycle of 7 distinct vertices? And why?

(c) Let say we remove node F and G from this graph and each edge is associated with a weight. What algorithm would you use to find the Hamiltonian cycle? And how does it work?

Problem 3 Probability + Counting ($3 \times 3 \times 3 = 27$ pts)

An urn A contains eleven labeled balls, labels 1,2, \dots , 11. An urn B contains six labeled balls, labels 1,2, \dots , 6.

(a) Two balls are drawn, one from A and one from B. What is the sample space? What is the probability that the sum of the labels on the balls is odd? What is the probability that the sum of the labels on the balls is 9?

(b) Two balls are drawn one after the other without replacement and the order matters from urn A. What is the sample space? What is the probability that the sum of the labels on the balls is odd and the first ball number must be greater than the 2nd ball number)? What is the probability that the sum of the labels on the balls is 9?

(c) Two balls are drawn from urn B one after the other with replacement and the order matters. What is the sample space? What is the probability that the sum of the labels on the balls is even? What is the probability that the sum of the labels on the balls is 10?

Problem 4 Graph: 7 + 12 pts

For a graph $G = (V, E)$, let $d(v)$ be the degree of the vertices $v \in V$.

(a) A complete graph CG $G = (V, E)$ is a graph where each node v has one edge to each of all other nodes. Given $|V| = n$, that means $d(v) = n - 1$, what is $|E|$?

(b) Please prove (a) via induction where the base case is when $n = 2$.