CS 528: Quantum Computing

Assessment Exam

Date: 04/15/2019

Instructions:
I leave plenty of space on each page for your computation. One extra blank sheet is also attached at the end of this exam if you need more computation space. If your answer is incorrect but you show the computation process, then partial credits will be given. Use your time wisely and good luck.

First Name:

Last Name:

Group ID:

Score: **/120**

Problem 1 Gram-Schmidt Orthonormalization:(5+10) pts

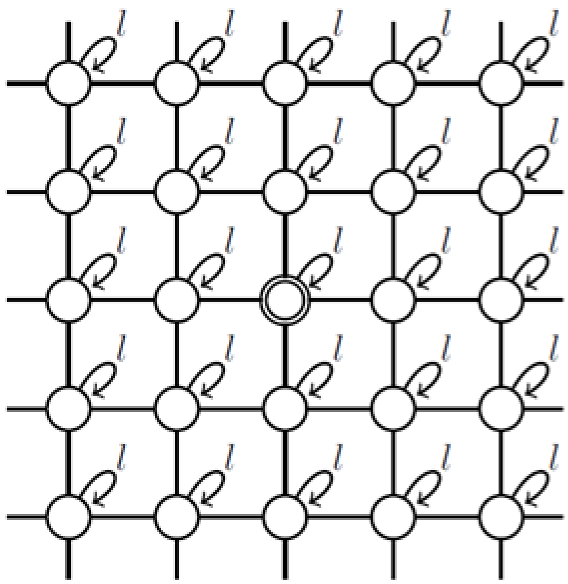
Let b be a basis that $b_1 = (-1, 0, 1), b_2 = (2, -1, 1)$ and $b_3 = (1, 3, -1)$.
(a) Is b an orthonormal basis? If not, which condition does it violate?

(b) Use the Gram-Schmidt procedure to find the corresponding (x_1, x_2, x_3), and (y_1, y_2, y_3).

Problem 2 Lackadaisical Walk on 2D: 15 pts

We learned about the quantum walk with the coin operator as the regular Hadamard operator and we know that the spreading speed (wrt to amplitudes), QW has the advantage. However, it remains an open question for QW on 2D and 1D. One of the approach for boosting 2D is by using the Lackdaisical walk. Please describe the coin operator and simulate the first step of the walk where variable $l = 5$ where [**Note: coin operator is changed, different from homework**]

Unitary Operator : $U = S(I \otimes C)$
Coin Operator $C = I - 2|s_c\rangle\langle s_c|$,
Initial Coin State: $|s_c\rangle = \frac{1}{\sqrt{4+5}}(|\uparrow\rangle + |\downarrow\rangle + |\rightarrow\rangle + |\leftarrow\rangle + \sqrt{5}|\circlearrowright\rangle)$
Initial System State: $|\psi_0\rangle = \frac{1}{\sqrt{N}}\sum_{v=1}^{N}|v\rangle \otimes |s_c\rangle$
And it can be visualized (adding self loop) as the following:



Problem 3 Analysis Technique Proof: 15 pts

Show that $e^{iAx} = cos(x)I + isin(x)A$ where $A^2 = I$ and x is some real number.

Problem 4 Simon's Algorithm: 15 pts

Run Simon's algorithm on the following input x (with $N = 8$):
$x_{000} = x_{111} = 000$ $x_{001} = x_{110} = 001$
$x_{010} = x_{101} = 010$ $x_{011} = x_{100} = 011$
We notice $x_i = x_{i \oplus 101}$ for all $i \in \{0,1\}^3$, so $s = 111$.
(a) Give the state after measuring the second register (the measurement gave $|011\rangle$).

(b) Use $H^{\otimes n}|i\rangle = \frac{1}{\sqrt{2}} \sum_{j \in \{0,1\}^n} (-1)^{i \cdot j} |j\rangle$, give the state after the final Hadamard.

(c) If s was 111, then after two runs we obtain $j = 011$ and $j = 101$. With 2 runs, we can determine s. Why?

Problem 5 Shor's Algorithm: 5x3 = 15pts

Use Shor's algorithm to find the period of the function $f(x) = 5^x \bmod 12$ by using a Fourier transform over $q = 256$ (in another word, Reg1 has 8 qubits and it is obvious $N^2 = 12^2 < q = 2^8 = 256 \leq 2N^2$). Write down all intermediate states (including both Reg 1 and Rge 2) of the quantum part of Shor's algorithm (the state after the firts Fourier transform $|\psi_1\rangle$, the state after the oracle function$|\psi_2\rangle$, the state after the observation $|\psi_3\rangle$, the state after the inverse fourier transform $|\psi_4\rangle$, the state after measurement $|\psi_5\rangle$).

Problem 6 Entanglement: 5 + 5 pts

(a) Draw the circuit for generating one of the Bell states $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ and please verify your circuit.

(b) What message would be sent when we use the superdense coding on the state described in (a)? Verify your answer.

Problem 7 Deutsch, Deutsch Josza Algorithms: 5 + 10 pts

In the Deutsch and Deutsch-Jozsa algorithms, when we consider U_f as a single-qubit operator $\hat{U}_{f(x)}$, $\frac{|0\rangle - |1\rangle}{\sqrt{2}}$ is an eigenstate of $\hat{U}_{f(x)}$, whose associated eigenvalue gives us the answer to the Deutsch problem. Suppose we did not prepare $\frac{|0\rangle - |1\rangle}{\sqrt{2}}$ but $|\mathbf{0}\rangle$ instead in the target qubit and we just run the same circuit on that configuration.
(a) Please compute and explain the probability that we get the right answer for the Deutsch algorithm circuit.

(b) Same as (a) but now for Deutsch-Jozsa algorithm

Problem 8 Grover's Algorithm: 5 + 5 pts

In $\{|G\rangle, |B\rangle\}$ basis, we may write the Grover iteration as

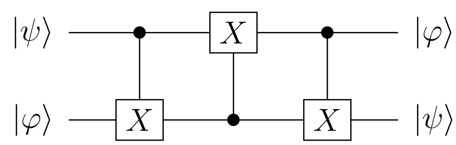
$$G = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

(b-1) Given sample space Ω where $|\Omega| = 2^8$ and let $Sx = \{x | f(x) = 1 \wedge x \in \Omega\}$. Let say $|S_x| = 6$ and you run Grover in order to find the possible solutions. Please write out the Grover operator in real numbers. [Hint: Be careful as each Grover roation rotates by 2*the original degree]

(b-2) What is the number of required invocations of Grover operator? (Be precise to the **2nd digit** after the decimal point).

Problem 9 SWAP: 5*2 = 10 pts

(a) Via matrix multiplication, show that the following circuit implements **Swap** operator. X is the NOT gate.



(b) Find a two-qubit unitary U such that $U^2 = SWAP$

Problem 10 Scratch Paper Area 1

Use this sheet if you need extra space. **DO NOT** detach this sheet from the exam.

Problem 11 Scratch Paper Area 2

Use this sheet if you need extra space. **DO NOT** detach this sheet from the exam.