CS 528: Quantum Computation
Problem Set 1+2

MW: 2:00 - 3:15 pm

Out: 02/20/2019 Due: 02/27/2019

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. Please directly hit the point when solving a problem. Cumbersome description might receive fewer credits, even it is correct. If your answer is incorrect but you your logic is on the right track, then partial credits will be given. Please staple your solution and use the space wisely.

First Names:

Group ID:

Score: **/50+100**

Problem 1 2-Qubit teleport: 50 pts

We briefly went over the paper "Probabilistic Teleportation of Arbitrary Two-Qubit Quantum State via Non-Symmetric Quantum Channel"[1]. Please summarize this procotol and discuss why at step 2 that if the result is$|0\rangle_4$(denoted by $m_4 = 0$), the original state can not be reconstructed and the teleportation fails.

[1] https://www.mdpi.com/1099-4300/20/4/238

Problem 2 Gram-Schmidt Orthonormalization:(5+10) pts

Let b be a basis that $b_1 = (-1, 0, 2), b_2 = (4, -1, 1)$ and $b_3 = (1, 1, -1)$.
(a) Is b an orthonormal basis? If not, which condition does it violate?

(b) Use the Gram-Schmidt procedure to find the corresponding (x_1, x_2, x_3), and (y_1, y_2, y_3).

Problem 3 Schroedinger Equation: 10 pts

When we have $|\psi(t)\rangle = U|\psi(0)\rangle$, where $U = e^{\frac{-iHt}{\hbar}}$, then $|\psi(t)\rangle$ is a solution to the Schroedinger equation

Problem 4 Simple Computation: 20 pts

Suppose we are given:

$$T = |1\rangle\langle 1| \otimes (|+\rangle\langle +| - |-\rangle\langle -|) + |0\rangle\langle 0| \otimes (|+\rangle\langle +| + |-\rangle\langle -|)$$

where $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. Let S be a swap gate.
(a) Show $\{|+\rangle, |-\rangle\}$ is an orthonormal basis

(b) Find another orthonormal basis and prove it

(c) Show that T is CNOT operator

(d) Give the matrix representation of TS (S applied first then T).

Problem 5 Entanglement: 5 +5+5pts

(a) Please describe the EPR paradox.

(b) Draw the circuit for generating one of the Bell states $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ and please verify your circuit.

(c) Please show the message 11 is sent when we use the superdense coding on the state described in (b).

Problem 6 Orthonormal Basis + Inner Product: 5+5 pts

(a)Please determine if the following is orthonormal basis $\{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix}\}$

(b) Show the result of $\langle 01|\alpha\beta\rangle$ where $|\alpha\rangle = \frac{\sqrt{3}}{\sqrt{12}}|0\rangle + \frac{3}{\sqrt{12}}|1\rangle$, $|\beta\rangle = \frac{3}{\sqrt{11}}|0\rangle - \frac{\sqrt{2}}{\sqrt{11}}|1\rangle$

Problem 7 Deutsch, Deutsch Josza Algorithms: 20pts

In the Deutsch and Deutsch-Jozsa algorithms, when we consider U_f as a single-qubit operator $\hat{U}_{f(x)}$, $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$ is an eigenstate of $\hat{U}_{f(x)}$, whose associated eigenvalue gives us the answer to the Deutsch problem. Suppose we did not prepare $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$ but $|\mathbf{0}\rangle$ instead in the target qubit and we just run the same circuit on that configuration.
(a) Please compute and explain the probability that we get the right answer for the Deutsch algorithm circuit.

(b) Same as (a) but now for Deutsch-Jozsa algorithm

Problem 8 Classical approach for Simon's problem : 10 pts

Please show that the complexity (in terms of the oracle calls) for a classical approach is bounded from above by $O(\sqrt{2^n})$