MAT 115: Finite Math for Computer Science
Problem Set 1

Due: 02/13/2019

Instructions:
I leave plenty of space on each page for your computation. If you need more sheet, please attach your work right behind the corresponding problem. If your answer is incorrect but you show the computation process, then partial credits will be given. Please staple your solution and use the space wisely. If you cannot turn in the assignment by the deadline, late penalty will be applied. 10 percent reduction will be applied per day for the late assignment.

First Name:

Last Name:

Group ID:

Score: **/155**

Problem 1 Boolean Functions 5 pts (2+1+2)

Given a function $f : \{0,1\}^4 \to \{0,1\}$, please answer the following :
(a) Please show two different input instances from the domain.

(b) How many different outputs are there in the codomain?

(c) What is the number of possible boolean functions f?

Problem 2 Truth Table : 10pts

Make a truth table for $(((p \to q) \wedge (\sim p \oplus r)) \vee (q \oplus r)) \wedge \sim (q \vee \sim r)$
(hint: $p \to q = \sim p \vee q$)

Problem 3 Algebraic Rules: 10 pts

Is the function $((\sim p \vee q) \wedge (p \vee \sim r)) \wedge (\sim p \vee \sim q)$ equal to the function $\sim (p \vee r)$?

Problem 4 Algebraic Rules: 10 pts

Is the boolean function $(r \vee p) \wedge (\sim r \vee (p \wedge q)) \wedge (r \vee q)$ equal to the function $p \wedge q$?

Problem 5 Algebraic Rules: 10 pts

Is the boolean function$\sim ((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (p \wedge q)$ equal to the function $\sim p$?

Problem 6 Statements: 5+5+5pts

Practice in symbols: Eg. if my average is over 97, then I get an A+ in my grade. Simply let p = my average is over 97, q = I get an A+ grade in my grade. Then we know $p \to q$. Please express the statements in symbols, and then the converse, contrapositive and inverse of these conditional statements
(a) If it snows today, I will ski tomorrow.

(b) If there is a quiz, then I will come to the recitation on Friday

(c) If a positive integer has no divisors other than 1 and itself, then it is a prime.

Problem 7 Logic Reasoning: 10pts

Explain, without using a truth table, why $f = (p \vee \sim q) \wedge (q \vee \sim r) \wedge (r \vee \sim p)$ is true when p, q, and r have the same truth value and it is false otherwise.

Problem 8 Base Change : 15 (5+5+5)pts

(a) Please convert 20190211_{10} into a base 16 number. You must show the computation

(b) Please convert $ABCDE0E_{16}$ into a base 10 number.

(c) Please show the result of $20190211_{10} + ABCDE0E_{16}$ in base 13.

Problem 9 Circuit Design: 10 + 5 pts

1) Please design the circuit for the NXOR (NOT-XOR) gate by using AND, NOT and OR gate (NOT-XOR evaluates inputs (x, y) to one only when $x = y$; otherwise the output is 0).

2) Please verify your circut with the four possible inputs by showing the ouputs after each gate and then the final output.

Problem 10 Circuit Design: 10pts

Construct a combinatorial circuit using NOT gates, OR gates and AND gates that produces the output $((\sim p \vee \sim r) \wedge (\sim q \oplus q)) \vee (\sim p \wedge (q \vee r))$ from input bits p, q, and r. Please make sure your circuit is as short as possible.

Problem 11 Circuit: 10pts

Please explain why the full adder has OR at the end, instead of another half adder. Also, is it OK to replace that OR gate with XOR gate? Why?

Problem 12 Proposition Equivalence : 5+10 pts

(a) Show that $((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology (always evaluates to truth) by using a truth table

(a) Show that $((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology (always evaluates to truth) by using algebraic rules (proposition equivalence rules)

Problem 13 Research: 10pts

We have been learning logic operators and those boolean functions. One type of well-studied boolean functions is 3-SAT. Please simply describe how a 3-SAT function is defined and its possible applications (maybe be good to read about NP-complete if you are interested in computational complexity).

Problem 14 Representing Function: 10pts

Given $f : \{0,1\}^3 \to \{0,1\}$, we can easily interpret it as $f(p,q,r) = s$ where $p,q,r \in \{0,1\}$ and $s \in \{0,1\}$. If we have $f(0,0,0) = 1, f(0,1,0) = 1, f(1,0,0) = 0, f(1,1,0) = 1, f(0,0,1) = 1, f(0,1,1) = 1, f(1,0,1) = 0$ and $f(1,1,1) = 1$. Please derive the boolean function f.